NON POSSO CONTROLLARE CIÒ CHE NON POSSO MISURARE

THEORY AND APPLICATIONS OF THE ANALYTIC HIERARCHY PROCESS (AHP)

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Decisioni

- Ogni giorno dobbiamo affrontare una quantità di **decisioni**, da quelle banali alle più serie.
- Le nostre decisioni non sempre provengono da motivazioni perfettamente **razionali**.
- All'origine di ogni decisione è presente una combinazione di **ragione** e di **emozione**.
  - L'**avversione** o la **propensione** al **rischio** sono attitudini originate nelle strutture del cervello che elaborano emozioni, es. la paura.

Teorie delle decisioni

- Una **teoria delle decisioni** si occupa dei procedimenti necessari per ottenere risultati ottimali.
- Se sentiamo il bisogno di analizzare, di essere coerenti, di far luce, di farci aiutare nello scegliere, la teoria può darci un **aiuto per giungere alla decisione**.
- Una teoria delle decisioni risponde prevalentemente a una esigenza di **razionalizzazione**.

Teorie delle decisioni

- Elementi del processo decisionale:
  - Come affrontare il problema?
  - Come analizzarlo?
  - Come descrivere le alternative?
  - Come misurare la portata delle azioni?
  - Quali sono le ricadute (conseguenze) delle scelte?
- Anche per dare risposte a questioni del genere, si sono sviluppate diverse teorie delle decisioni.

Decisioni e conoscenza

- La **conoscenza** è, ovviamente, fondamentale per il processo decisionale.
- Ci sono situazioni reali (es. gioco d'azzardo) che hanno indotto alla concettualizzazione e alla formalizzazione dell' **incertezza,** al suo **trattamento** e alla sua **misura**.

Un ambito di applicazione: il governo del territorio

- Il **territorio** è il risultato di una molteplicità di componenti che interagiscono tra loro:
  - economiche,
  - sociali,
  - infrastrutturali,
  - etc.
- Non si può prescindere da questa natura multidimensionale anche nell'attività di **valutazione** e **scelta di azioni** che, proprio per la loro varietà, richiedono un'attenta conoscenza degli effetti non solo nel contesto di riferimento, ma anche sugli altri ambiti del sistema urbano.

Pianificazione territoriale

- I processi di pianificazione territoriale sono, per loro natura, **complessi**, in quanto il
  - gli interessi e i soggetti coinvolti sono spesso
    - interagenti
    - conflittuali
    - variabili nel tempo

Sviluppo sostenibile del territorio

- “Lo sviluppo sostenibile è uno sviluppo che soddisfa i bisogni del presente senza compromettere la possibilità delle generazioni future di soddisfare i propri bisogni.”

Sviluppo sostenibile del territorio

- Secondo questo schema lo sviluppo sostenibile può essere visto come una combinazione della posizione
  - dell' economista
  - del sociologo
  - dell' ambientalista.
- Assumere delle scelte significherà, pertanto, riconoscere ed accettare delle priorità ed attraverso esse prediligere una posizione rispetto ad un' altra, stabilendo dei criteri

Le valutazioni

- La costruzione di scelte che hanno effetti nel lungo periodo è complessa e richiede un approccio strategico.
- Le **valutazioni** sono lo strumento critico con il quale si possono affrontare i problemi conseguenti alle conflittualità tra gli obiettivi dello sviluppo sostenibile e quindi dedurre le priorità tra opzioni alternative.

Multi-Criteria Decision Methods (MCDM)

- I MCDM nascono per affrontare problemi di **scelta** e di **ordinamento**
- essi hanno la funzione di fornire al decisore strumenti per affrontare problemi decisionali caratterizzati
  - da una molteplicità di punti di vista significativi
  - da un insieme di dati eterogenei di natura sia quantitativa che qualitativa

MCDM

- L'analisi multicriteriale è stata approfondita soprattutto a partire dagli **anni '80**, trovando molteplici ambiti di **applicazione** in contesti di decisione sia **individuali** che **collettivi**.
- Essa parte dal presupposto che in un contesto sociale il problema della determinazione di una soluzione "**ottima**" in un problema decisionale non può essere risolto utilizzando un solo criterio o un'unica funzione obiettivo.
- I Multi-Criteria Decision Methods rappresentano un superamento dei limiti di teorie consolidate quali l'utilità e la programmazione lineare.

MCDM

- In generale non esiste una decisione possibile (una soluzione del problema) che sia contemporaneamente la migliore da tutti i punti di vista ritenuti significativi per trattare un problema decisionale nella sua globalità: ogni alternativa può presentare **vantaggi e svantaggi** rispetto alle altre alternative.

Definizione di un problema

- Definire l' obiettivo "O" della decisione
- Individuare un insieme finito $A=\{A_1, A_2, \ldots A_n\}$ di alternative, o azioni, tra le quali il decisore deve operare la scelta
- $\{C_1, C_2, , C_k\}$di criteri che le alternative devono soddisfare al fine di realizzare l' obiettivo.
- La risoluzione del problema di decisione passa attraverso:
  - la valutazione delle **alternative rispetto ai criteri**
  - la determinazione dell' importanza dei **criteri per** il raggiungimento dell' **obiettivo**;
  - la valutazione globale, attraverso l'aggregazione delle valutazioni locali, delle **alternative rispetto all'obiettivo**

Tabella delle valutazioni

	$C_1(p_1)$	$C_h(p_h)$	$C_k(p_k)$
A_1	v_{11}	v_{1h}	v_{1k}
A_2	v_{21}	v_{2h}	v_{2k}
…	…	…	…
A_n	v_{n1}	v_{nh}	v_{nk}

- Aggregando opportunamente le valutazioni locali $v_{i1}, v_{i2}, \ldots, v_{ik}$, si determina la valutazione globale $V(A_i)$ dell'alternativa A_i
- le valutazioni globali cosi ottenute permettono di ordinare le alternative e quindi di operare la scelta
- A_i è preferita a A_j rispetto al criterio C_h

$$(A_i \succ A_j) \Leftrightarrow v_{ih} > v_{jh}$$

The weighted sum model (WSM)

	$C_1(p_1)$	$C_h(p_h)$	$C_k(p_k)$
A_1	v_{11}	v_{1h}	v_{1k}
A_2	v_{21}	v_{2h}	v_{2k}
…	…	…	…
A_n	v_{n1}	v_{nh}	v_{nk}

- $V_{WSM}(A_i) = p_1 v_{i1} + p_2 v_{i2} + \ldots + p_k v_{ik}$
- "single dimensional decision-making problems"
  - es. le alternative non possono essere delle auto valutate rispetto a criteri con unità di misura diverse, come costo (euro), consumo (km/litro), dimensioni (metro)
  - es. non si possono sommare euro e metri
  - $A_i \succ A_j \Leftrightarrow V_{WSM}(A_i) > V_{WSM}(A_j)$

WSM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WSM}(A_i) = p_1 v_{i1} + p_2 v_{i2} + p_3 v_{i3} + p_4 v_{i4}$

WSM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WSM}(A_i) = p_1 v_{i1} + p_2 v_{i2} + p_3 v_{i3} + p_4 v_{i4}$
  - $V_{WSM}(A_1) =$ 0.20 x 25 + 0.15 x 20 + 0.40 x 15 + 0.25 x 30= **21.50**

WSM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WSM}(A_i) = p_1 v_{i1} + p_2 v_{i2} + p_3 v_{i3} + p_4 v_{i4}$
  - $V_{WSM}(A_1)$ = 0.20 x 25 + 0.15 x 20 + 0.40 x 15 + 0.25 x 30= **21.50**
  - $V_{WSM}(A_2)$ = 0.20 x 10 + 0.15 x 30 + 0.40 x 20 + 0.25 x 30= **22**
  - $V_{WSM}(A_3)$ = 0.20 x 20 + 0.15 x 10 + 0.40 x 30 + 0.25 x 10= **20**

WSM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WSM}(A_i) = p_1 v_{i1} + p_2 v_{i2} + p_3 v_{i3} + p_4 v_{i4}$
  - $V_{WSM}(A_1) =$ 0.20 x 25 + 0.15 x 20 + 0.40 x 15 + 0.25 x 30= **21.50**
  - $V_{WSM}(A_2) =$ 0.20 x 10 + 0.15 x 30 + 0.40 x 20 + 0.25 x 30= **22**
  - $V_{WSM}(A_3) =$ 0.20 x 20 + 0.15 x 10 + 0.40 x 30 + 0.25 x 10= **20**

$$A_2 \succ A_1 \succ A_3$$

The weighted product model (WPM)

	$C_1(p_1)$	$C_h(p_h)$	$C_k(p_k)$
A_1	v_{11}	v_{1h}	v_{1k}
A_2	v_{21}	v_{2h}	v_{2k}
…	…	…	…
A_n	v_{n1}	v_{nh}	v_{nk}

- $$V_{WPM}\left(\frac{A_s}{A_t}\right) = \prod_{j=1}^{N}\left(\frac{v_{sj}}{v_{tj}}\right)^{p_j}$$
- Elimino le unità di misura
- "single and multi-dimensional decision-making problems"
- $$A_s \succ A_t \Leftrightarrow V_{WPM}\left(\frac{A_s}{A_t}\right) > 1$$

WPM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WPM}(A_s/A_t) = (v_{s1}/v_{t1})^{p1} (v_{s2}/v_{t2})^{p2} (v_{s3}/v_{t3})^{p3} (v_{s4}/v_{t4})^{p4}$

WPM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WPM}(A_s/A_t) = (v_{s1}/v_{t1})^{p1} (v_{s2}/v_{t2})^{p2} (v_{s3}/v_{t3})^{p3} (v_{s4}/v_{t4})^{p4}$
  - $V_{WPM}(A_1/A_2) = (25/10)^{0.20} (20/30)^{0.15} (15/20)^{0.40} (30/30)^{0.25} =$ **1.007**

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WPM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WPM}(A_s/A_t) = (v_{s1}/v_{t1})^{p1} (v_{s2}/v_{t2})^{p2} (v_{s3}/v_{t3})^{p3} (v_{s4}/v_{t4})^{p4}$
  - $V_{WPM}(A_1/A_2) = (25/10)^{0.20} (20/30)^{0.15} (15/20)^{0.40} (30/30)^{0.25} =$ **1.007**
  - $V_{WPM}(A_1/A_3) = (25/20)^{0.20} (20/10)^{0.15} (15/30)^{0.40} (30/10)^{0.25} =$ **1.157**
  - $V_{WPM}(A_2/A_3) = (10/20)^{0.20} (30/10)^{0.15} (20/30)^{0.40} (30/10)^{0.25} =$ **1.149**

WPM
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WPM}(A_s/A_t) = (v_{s1}/v_{t1})^{p1} (v_{s2}/v_{t2})^{p2} (v_{s3}/v_{t3})^{p3} (v_{s4}/v_{t4})^{p4}$
  - $V_{WPM}(A_1/A_2) = (25/10)^{0.20} (20/30)^{0.15} (15/20)^{0.40} (30/30)^{0.25}$ = **1.007**
  - $V_{WPM}(A_1/A_3) = (25/20)^{0.20} (20/10)^{0.15} (15/30)^{0.40} (30/10)^{0.25}$ = **1.157**
  - $V_{WPM}(A_2/A_3) = (10/20)^{0.20} (30/10)^{0.15} (20/30)^{0.40} (30/10)^{0.25}$ = **1.149**

$$A_1 \succ A_2 \succ A_3$$

WPM (variante)

	$C_1(p_1)$	$C_h(p_h)$	$C_k(p_k)$
A_1	v_{11}	v_{1h}	v_{1k}
A_2	v_{21}	v_{2h}	v_{2k}
…	…	…	…
A_n	v_{n1}	v_{nh}	v_{nk}

- $$V_{WPMv}(A_i) = \prod_{j=1}^{N} (v_{ij})^{p_j}$$

- $$A_i \succ A_j \Leftrightarrow V_{WPMv}(A_i) > V_{WPMv}(A_j)$$

WPM (variante)
Esempio

	$C_1(0.20)$	$C_2(0.15)$	$C_3(0.40)$	$C_4(0.25)$
A_1	25	20	15	30
A_2	10	30	20	30
A_3	20	10	30	10

- $V_{WPMv}(A_i) = v_{i1}^{p1}\, v_{i2}^{p2}\, v_{i3}^{p3}\, v_{i4}^{p4}$
  - $V_{WPM}(A_1) = 25^{0.20}\ 20^{0.15}\ 15^{0.40}\ 30^{0.25} =$ **20.63**
  - $V_{WPM}(A_2) = 10^{0.20}\ 30^{0.15}\ 20^{0.40}\ 30^{0.25} =$ **20.48**
  - $V_{WPM}(A_3) = 20^{0.20}\ 10^{0.15}\ 30^{0.40}\ 10^{0.25} =$ **17.83**

$$A_1 \succ A_2 \succ A_3$$

Applicazione allo sviluppo sostenibile

- C1 Creazione di oasi di verde
- C2 Creazione di wifi free
- C3 Creazione di nuovi posti di lavoro

Due progetti urbani:

- A1
- A2

Applicazione allo sviluppo sostenibile

	0,3	0,2	0,5
	C1 (m2)	C2 (Mb)	C3 (numero)
A1	2000	20	1000
A2	3000	8	800

WPM

v(A1/A2) 1,189092474

v(A2/A1) 0,840977486

A1 > A2

punteggi su scala da 1 a 10

	0,3	0,2	0,5
	C1	C2	C3
A1	2	10	10
A2	3	4	8

WSM

7,6

5,7

A1 > A2

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Tabella delle valutazioni

- Rappresentazione di un problema multi-criterio attraverso la tabella delle valutazioni

	$C_1(p_1)$	$C_h(p_h)$	$C_k(p_k)$
A_1	v_{11}	v_{1h}	v_{1k}
A_2	v_{21}	v_{2h}	v_{2k}
…	…	…	…
A_n	v_{n1}	v_{nh}	v_{nk}

- Aggregando opportunamente le **valutazioni locali** $v_{i1}, v_{i2}, \ldots, v_{ik}$, si determina la **valutazione globale** $V(A_i)$ dell'alternativa A_i, le valutazioni globali cosi ottenute permettono di ordinare le alternative e quindi di operare la scelta.

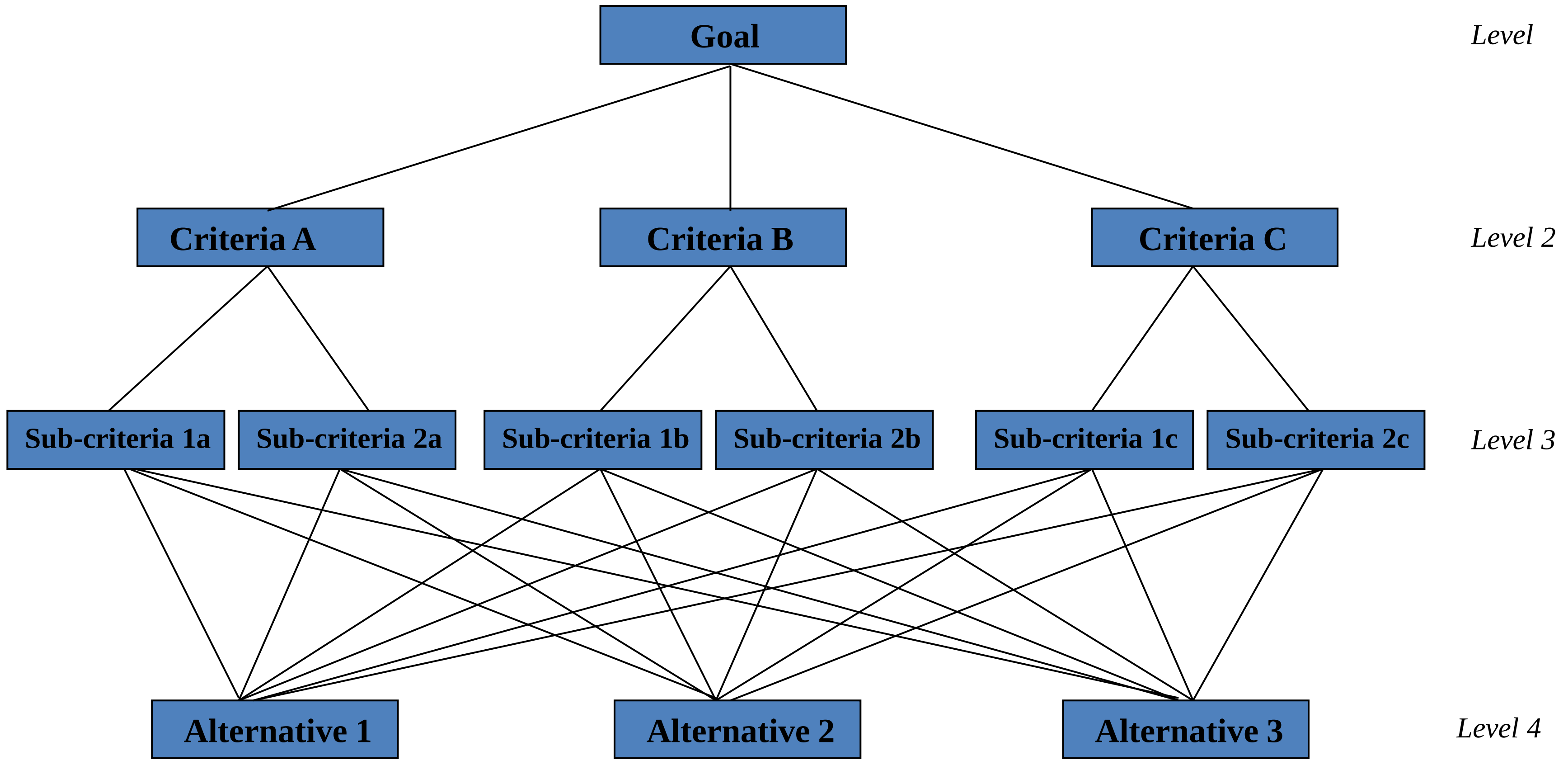
Matrici di confronto a coppie

- Una tecnica per determinare le **valutazioni locali** delle alternative rispetto ai criteri ed i pesi dei criteri rispetto all'obiettivo è rappresentato dalle **matrici di confronto a coppie**:
  - le alternative (risp. i criteri) vengono confrontate a 2 a 2 (attraverso un **rapporto di preferenza**) rispetto ad ogni singolo criterio (risp. obiettivo) per poi determinare un ranking pesato delle alternative (risp. dei criteri) rispetto ad un criterio fissato (risp. obiettivo).
- Tale tecnica è propria dell'**Analytic Hierarchy Process** (AHP).

AHP

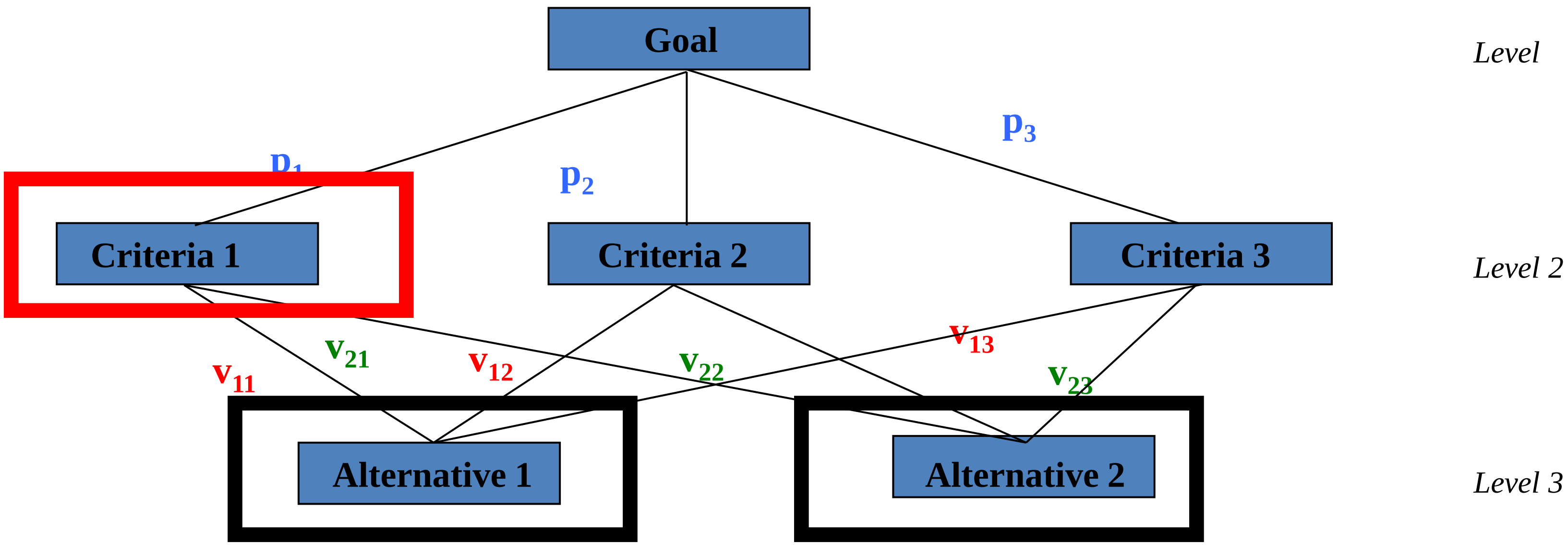
- The Analytic Hierarchy Process (AHP) is a multi-criteria decision making method, first introduced by T.L. Saaty (Saaty, 1980 and 1994) in the 70s. The principles of the AHP are logic, comprehensive, and it can be used in both quantitative and qualitative multi-criteria decision making problems.

AHP


Goal
Level
Criteria A
Criteria B
Criteria C
Level 2
Sub-criteria 1a
Sub-criteria 2a
Sub-criteria 1b
Sub-criteria 2b
Sub-criteria 1c
Sub-criteria 2c
Level 3
Alternative 1
Alternative 2
Alternative 3
Level 4

AHP

	$C_1(p_1)$	$C_2(p_2)$	$C_3(p_3)$
A_1	v_{11}	v_{12}	v_{13}
A_2	v_{21}	v_{22}	v_{23}

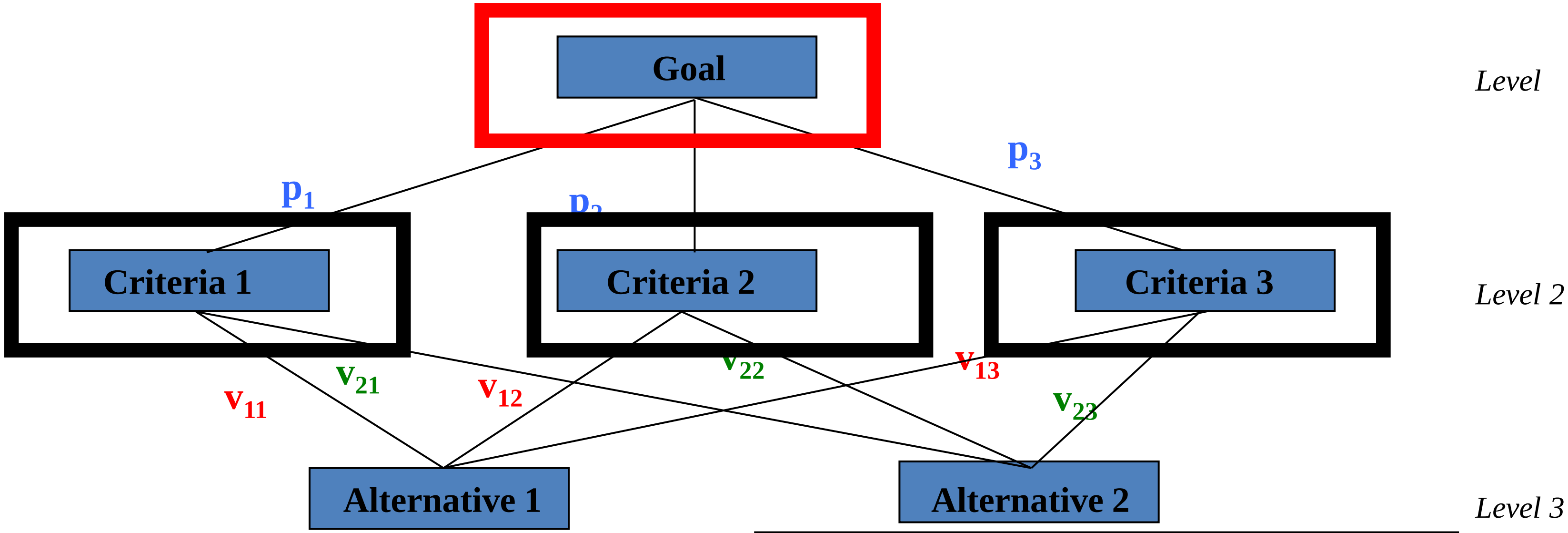


- $v(A_1) = p_1 v_{11} + p_2 v_{12} + p_3 v_{13}$
- $v(A_2) = p_1 v_{21} + p_2 v_{22} + p_3 v_{23}$

$$\begin{array}{c} \quad C_1 \quad A_1 \quad\quad A_2 \\ A = \begin{array}{c} A_1 \\ A_2 \end{array} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{array}{c} \to v_{11} \\ \to v_{21} \end{array} \end{array}$$

AHP

	$C_1(p_1)$	$C_2(p_2)$	$C_3(p_3)$
A_1	v_{11}	v_{12}	v_{13}
A_2	v_{21}	v_{22}	v_{23}



- $v(A_1) = p_1 v_{11} + p_2 v_{12} + p_3 v_{13}$
- $v(A_2) = p_1 v_{21} + p_2 v_{22} + p_3 v_{23}$

$$
\begin{array}{cc}
 & \begin{array}{cccc} G & C_1 & C_2 & C_3 \end{array} \\
C = \begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} & \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{array}{c} \rightarrow p_1 \\ \rightarrow p_2 \\ \rightarrow p_3 \end{array}
\end{array}
$$

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Saaty approach - Preference ratios

1	Equal Important
3	Weak Importance
5	Strong Importance
7	Very strong Importance
9	Absolute Importance
2,4,6,8	Intermediate values

Reciprocity property

$$a_{ji} = \frac{1}{a_{ij}} \; \forall \; i, j = 1, \ldots, n,$$

Example

$$A = \begin{pmatrix} 1 & 2 & 4 & 8 \\ \frac{1}{2} & 1 & 2 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & 1 & 5 \\ \frac{1}{8} & 2 & \frac{1}{5} & 1 \end{pmatrix}$$

Matrici di confronto a coppie

- Dato l'insieme delle alternative $A=\{A_1,A_2,\ldots,A_n\}$(risp. criteri), ad ogni coppia (A_i,A_j) è assegnato un numero reale positivo a_{ij} che esprime quanto A_i è preferita a A_j:

$$A = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \ldots & A_n \end{array} \\ \begin{pmatrix} 1 & a_{12} & \ldots & a_{1n} \\ a_{21} & 1 & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & 1 \end{pmatrix} \end{array}$$

$$a_{ij} > 1 \Leftrightarrow A_i \succ A_j$$

$$a_{ij} = 1 \Leftrightarrow A_i \sim A_j$$

$$a_{ij} < 1 \Leftrightarrow A_i \prec A_j$$

$$a_{ji} = \frac{1}{a_{ij}} \quad \forall i, j = 1,2,\ldots,n$$

$$a_{ii} = 1$$

AHP

$$A = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \ldots & A_n \end{array} \\ \begin{pmatrix} 1 & a_{12} & \ldots & a_{1n} \\ a_{21} & 1 & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & 1 \end{pmatrix} \end{array} \begin{array}{l} \\ \to v_{1i} \\ \to v_{2i} \\ \\ \to v_{ni} \end{array}$$

Weighting vector

eigenvector $A \times \underline{\hat{w}} = \lambda_{\max} \times \underline{\hat{w}}$

aritmetic mean $\underline{w}_{am} = \left(\frac{1}{n} \sum_j a_{1j}, \ldots, \frac{1}{n} \sum_j a_{nj} \right)$

geometric mean $\underline{w}_{gm} = \left(\sqrt[n]{a_{11} \times a_{12} \times \ldots \times a_{1n}} \;,\; \ldots.. \;,\; \sqrt[n]{a_{n1} \times a_{n2} \times \ldots \times a_{nn}} \right)$

Vettore delle priorità

- Determinazione delle priorità locali che esprimono **l'importanza relativa** degli elementi di un livello gerarchico rispetto ad ogni elemento del livello immediatamente superiore
- Es. alternative rispetto ad un fissato criterio C_i

$$A = \begin{matrix} & A_1 & A_2 & \dots & A_n \\ A_1 & 1 & a_{12} & \dots & a_{1n} \\ A_2 & a_{21} & 1 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ A_n & a_{n1} & a_{n2} & \dots & 1 \end{matrix} \quad \begin{matrix} \\ \to v_{1i} \\ \to v_{2i} \\ \\ \to v_{ni} \end{matrix}$$

Il vettore delle priorità $\underline{v} = (v_{1i}, v_{2i}, \dots, v_{ni})$ stabilisce un ordinamento di preferenza sull'insieme delle alternative in modo che $A_h \succ A_k \Leftrightarrow v_{hi} > v_{ki}$

Vettore media aritmetica

$$\underline{w}_{am} = \left(\frac{1}{n} \sum_j a_{1j}, \ldots, \frac{1}{n} \sum_j a_{nj} \right)$$

C	A_1	A_2	A_3
A_1	1	2	1
A_2	1/2	1	1/2
A_3	1	2	1

$$w_1 = \frac{1}{3}(1+2+1) = \frac{4}{3}$$

$$w_2 = \frac{1}{3}(1/2+1+1/2) = \frac{2}{3}$$

$$w_3 = \frac{1}{3}(1+2+1) = \frac{4}{3}$$

$$\Rightarrow \underline{w}_{am} = (4/3,\ 2/3,\ 4/3)$$

$$\underline{w}^* = \frac{\underline{w}}{\sum_{i=1}^{n} w_i}$$

$$w_1^* = \frac{4/3}{4/3+2/3+4/3} = \frac{4/3}{10/3} = \frac{4}{10} = \frac{2}{5}$$

$$w_2^* = \frac{2/3}{10/3} = \frac{1}{5}$$

$$w_3^* = \frac{4/3}{10/3} = \frac{2}{5}$$

$$\Rightarrow \underline{w}^* = (2/5,\ 1/5,\ 2/5)$$

Vettore media geometrica

$$\underline{w}_{gm} = \left(\sqrt[n]{a_{11} \times a_{12} \times \ldots \times a_{1n}} \;,\; \ldots\ldots \;,\; \sqrt[n]{a_{n1} \times a_{n2} \times \ldots \times a_{nn}} \right)$$

C	A_1	A_2	A_3
A_1	1	2	1
A_2	1/2	1	1/2
A_3	1	2	1

$$w_1 = \sqrt[3]{1 \times 2 \times 1} = \sqrt[3]{2} = 1.260$$

$$w_2 = \sqrt[3]{\frac{1}{2} \times 1 \times \frac{1}{2}} = \sqrt[3]{\frac{1}{4}} = 0.630$$

$$\Rightarrow \underline{w}_{gm} = (1.260, 0.630, 1.260)$$

$$w_3 = \sqrt[3]{1 \times 2 \times 1} = \sqrt[3]{2} = 1.260$$

$$\underline{w}^* = \frac{\underline{w}}{\sum_{i=1}^{n} w_i}$$

$$w_1^* = \frac{1.260}{3.150} = 0.4 = \frac{2}{5}$$

$$w_2^* = \frac{0.630}{3.150} = 0.2 = \frac{1}{5}$$

$$w_3^* = \frac{1.260}{3.150} = 0.4 = \frac{2}{5}$$

$$\Rightarrow \underline{w}^* = (2/5,\ 1/5,\ 2/5)$$

Sintesi - Passi AHP

- Costruzione della **struttura gerarchica** nei cui livelli vanno disposti gli elementi in gioco nel problema decisionale
- Raccolta dei dati per la determinazione dei **rapporti di preferenza** degli elementi di un livello rispetto ad un qualsiasi elemento del livello superiore che funge da criterio
- Stima dei **pesi relativi** o **valutazioni locali** degli elementi di un livello rispetto agli elementi del livello superiore
- **Aggregazione** dei pesi relativi degli elementi di ogni livello per giungere ad un ordinamento (**ranking**) pesato delle alternative.

Consistent PCMs

Consistency

$$a_{ik} = a_{ij} \cdot a_{jk} \quad \forall\, i, j, k = 1, \ldots, n.$$

Example

$$A = \begin{pmatrix} 1 & 1 & \underline{2} & \underline{\underline{8}} \\ 1 & 1 & 2 & 6 \\ \frac{1}{2} & \frac{1}{2} & 1 & \underline{3} \\ \frac{1}{8} & \frac{1}{6} & \frac{1}{3} & 1 \end{pmatrix}$$

NOT CONSISTENT

Consistent PCM and consistent vectors

$$A = (a_{ij}) \text{ is consistent } \Leftrightarrow \exists \underline{w} = (w_1, \ldots, w_n) : \frac{w_i}{w_j} = a_{ij} \quad \forall\, i, j.$$

Example

$$A = \begin{pmatrix} 1 & 2 & 6 \\ \frac{1}{2} & 1 & 3 \\ \frac{1}{6} & \frac{1}{3} & 1 \end{pmatrix} \qquad \underline{w} = k \cdot (1, \frac{1}{2}, \frac{1}{6})$$

Thus, if $A = (a_{ij})$ is a consistent PCM, then it is reasonable to choose a weighting vector in the set of consistent vectors, while if $A = (a_{ij})$ is an inconsistent PCM then we look for a vector that is close to be a consistent vector.

Ordinamento delle preferenze

Se la matrice è consistente

$$\underline{w}_{\lambda_{\max}} \qquad \underline{w}_{am} \qquad \underline{w}_{gm}$$

sono proporzionali ed indicano lo stesso ordinamento delle preferenze

$$w_{i_1} > w_{i_2} > \ldots > w_{i_n} \qquad \text{Þ} \qquad A_{i_1} \succ A_{i_2} \succ \ldots \succ A_{i_n}$$

Vettori consistenti (esempio)

C	A_1	A_2	A_3
A_1	1	2	1
A_2	1/2	1	1/2
A_3	1	2	1

⇒

$$\underline{w}_{am} = (4/3,\ 2/3,\ 4/3)$$

$$\underline{w}_{gm} = (1.260, 0.630, 1.260)$$

⇒ Sono vettori consistenti e forniscono lo stesso ordinamento

A è consistente

Applicazione AHP

O	C_1	C_2	C_3
C_1	1	2	4
C_2	1/2	1	2
C_3	1/4	1/2	1

$$\underline{p} = \underline{w}^*_{am} = (2.33, 1.17, 0.58)/4.08$$

$$\underline{p} = \underline{w}^*_{gm} = (2, 1, 0.5)/3.50$$

Priorità locali

$p_1 = 0.57$

$p_2 = 0.29$

$p_3 = 0.14$

C1	A1	A2
A1	1	1/2
A2	2	1

$$(v_{11}, v_{21}) = (0.33, 0.67)$$

C2	A1	A2
A1	1	4
A2	1/4	1

$$(v_{12}, v_{22}) = (0.8, 0.2)$$

C3	A1	A2
A1	1	5
A2	1/5	1

$$(v_{13}, v_{23}) = (0.83, 0.17)$$

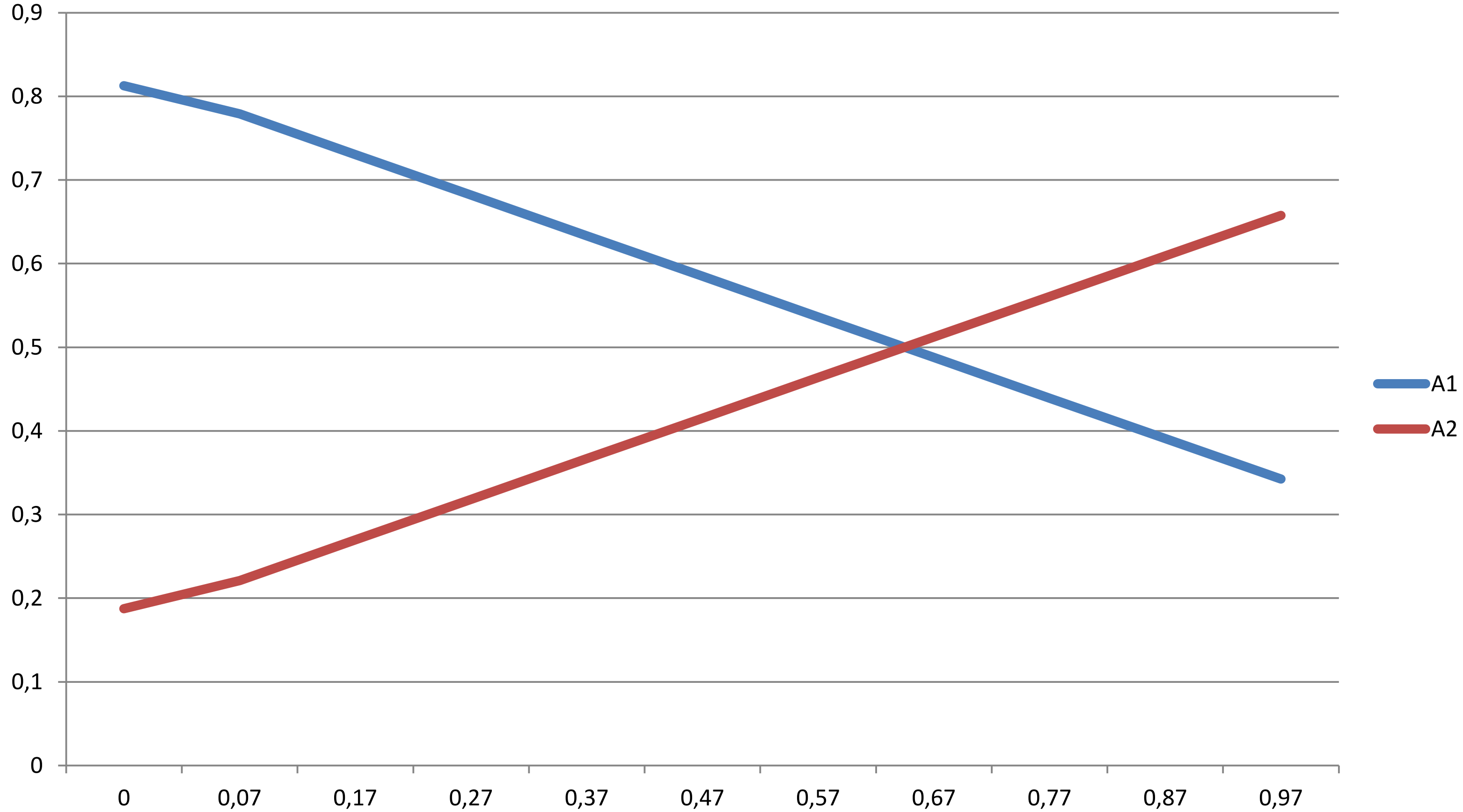
	$C_1(0.57)$	$C_2(0.29)$	$C_3(0.14)$
A_1	0.33	0.8	0.83
A_2	0.67	0.2	0.17

Priorità globali

- $v(A_1) = p_1 v_{11} + p_2 v_{12} + p_3 v_{13} = 0.57*0.33 + 0.29*0.8 + 0.14*0.83 = 0.54$
- $v(A_2) = p_1 v_{21} + p_2 v_{22} + p_3 v_{23} = 0.57*0.67 + 0.29*0.2 + 0.14*0.17 = 0.46$

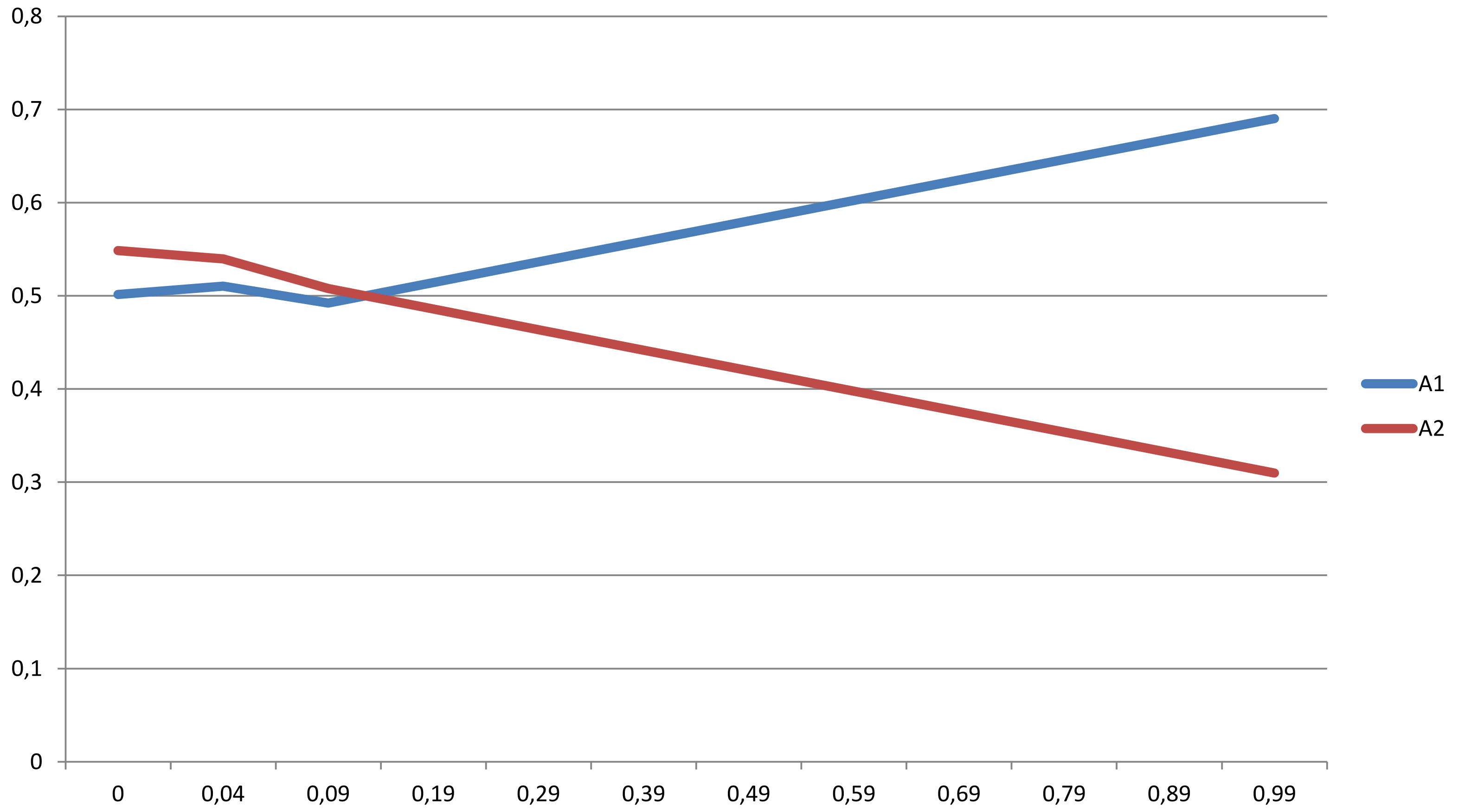
$$A_1 \succ A_2$$

Analisi di sensitività



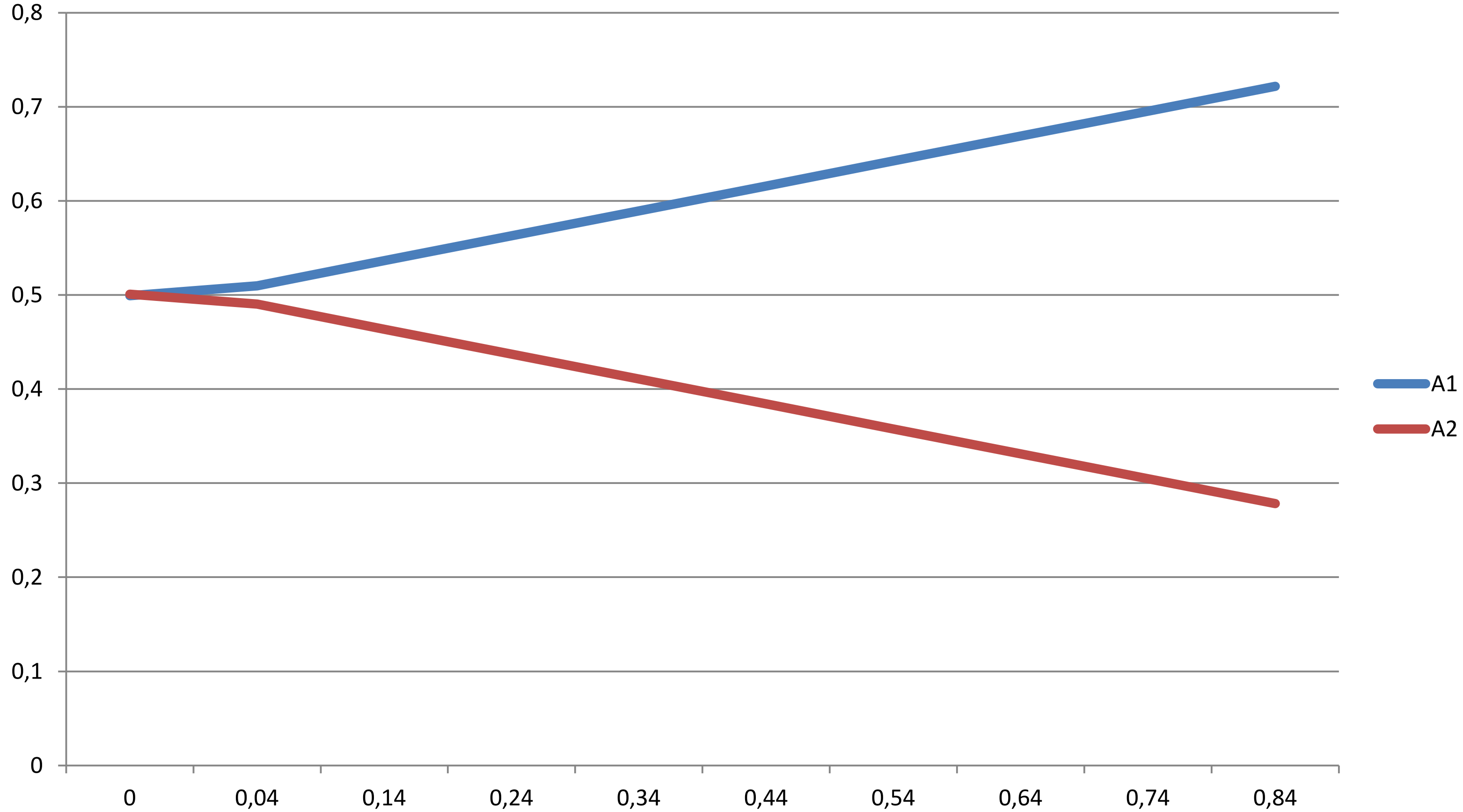
$$p_1 = 0.57$$

Analisi di sensitività



$$p_2 = 0.29$$

Analisi di sensitività



$p_3 = 0.14$

Consistency index

Since the decision maker is often not able to express consistent preferences in case of several criteria, Saaty proposes a measure of the consistency of a PCM:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

The right eigenvector $\underline{w}_{\lambda_{max}}$ corresponding to the maximum eigenvalue λ_{max} is chosen as a weighting vector for the alternatives.

Additive approach

$$a_{ij} \in [-a,\ a] \quad a \in R$$

Additive Reciprocity

$$a_{ji} = -\ a_{ij}$$

Additive Consistency and consistent vectors

$$a_{ik} = a_{ij} + a_{jk} \Leftrightarrow \exists\ \underline{w} : w_i - w_j = a_{ij}$$

Example

$$A = \begin{pmatrix} 0 & 2 & 5 \\ -2 & 0 & 3 \\ -5 & -3 & 0 \end{pmatrix} \qquad \underline{w} = (5, 3, 0)$$

Fuzzy approach

$$a_{ij} \in]0,1[$$

Fuzzy Reciprocity

$$a_{ji} = 1 - a_{ij}$$

Multiplicative Fuzzy Consistency and consistent vectors

$$a_{ik} = \frac{a_{ij}a_{jk}}{a_{ij}a_{jk}+(1-a_{ij})(1-a_{jk})} \Leftrightarrow \exists \underline{w} : \frac{w_i(1-w_j)}{w_i(1-w_j)+(1-w_i)w_j} = a_{ij}$$

Example

$$A = \begin{pmatrix} 0.5 & 0.6 & 0.\overline{7} \\ 0.4 & 0.5 & 0.7 \\ 1-0.\overline{7} & 0.3 & 0.5 \end{pmatrix} \qquad \underline{w} = (0.6, 0.5, 0.3)$$

The shortcomings - Difficulty to be consistent

- **Multiplicative case.**

  $a_{ij} \in S = \{\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

  $a_{12} = 5, \ a_{23} = 3$

  $a_{12} \cdot a_{23} = 15 \notin S$

- **Additive case**

  $a_{ij} \in [-a, a]$

  $a_{12} = \frac{1}{2}a, \ a_{23} = \frac{3}{4}a$

  $a_{12} + a_{23} = \frac{5}{4}a \notin [-a, a]$

The shortcomings - Consistency index for a multiplicative PCM

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

- CI is not easy to compute
- CI has not a simple and geometric meaning
- there is not a corresponding CI for additive and fuzzy PCMs

The shortcomings - dependence of scale inversion condition

By reciprocity, the information given by $A = (a_{ij})$ has to be equivalent to the information provided by A^T.

Example

$$A = \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ 4 & 1 & 2 & 1 \\ 2 & \frac{1}{2} & 1 & \frac{1}{4} \\ 2 & 1 & 4 & 1 \end{pmatrix} \qquad a_{14} \neq a_{13}a_{34}$$

- $w_{\lambda_{max}}(A) = (0.20177, 0.64411, 0.28131, 0.68211)$,
  $w_{\lambda_{max}}(A^T) = (0.76963, 0.22028, 0.55202, 0.23327)$.
- $w_{\lambda_{max}}(A)$ and $w_{\lambda_{max}}(A^T)$ provide different rankings:

$$x_4 \succ x_2 \succ x_3 \succ x_1,$$

$$x_2 \succ x_4 \succ x_3 \succ x_1,$$

Index

- Theory
  - Multi-criteria methods
  - AHP
  - An algebraic structure for Pairwise Comparison Matrices (PCMs)
- Applications
  - PCMs and Bayesian network for Data Privacy
  - AHP for sustainable development
  - AHP for energetic-environmental requalification
  - Dynamic management of a Portfolio
  - Management of human resources

PCMs on real divisible alo-groups

- $X = \{x_1, x_2, ...x_n\}$ is a set of alternatives
- $\mathcal{G} = (G, \odot, \leq)$ is a real continuous divisible alo-group
- $A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ defined on $\mathcal{G} \Leftrightarrow a_{ij} \in G$

Alo-group

Definition

Let G be a non empty set, $\odot : G \times G \to G$ a binary operation on G, $\leq$ a total weak order on G. Then $\mathcal{G} = (G, \odot, \leq)$ is an *abelian linearly ordered group* (*alo-group*), if and only if $(G, \odot)$ is an abelian group and

$$a \leq b \Rightarrow a \odot c \leq b \odot c.$$

- e its *identity*
- $a^{(-1)}$ the *inverse* of $a \in G$ with respect to $\odot$
- $\div$ the *inverse operation* of $\odot$, defined by:

$$a \div b = a \odot b^{(-1)} \quad \forall a, b \in G.$$

Isomorphism

Definition

An *isomorphism* between two alo-groups $\mathcal{G} = (G, \odot, \leq)$ and $\mathcal{G}' = (G', \circ, \leq)$ is a bijection $h : G \to G'$ that is both a lattice isomorphism and a group isomorphism, that is:

$$x < y \Leftrightarrow h(x) < h(y) \quad \textit{and} \quad h(x \odot y) = h(x) \circ h(y)$$

G-distance

Definition

The operation

$$d : (a, b) \in G \times G \mapsto d(a, b) \in G$$

is a $\mathcal{G}$-*metric* or $\mathcal{G}$-*distance* if and only if:

1. $d(a, b) \geq e$;
2. $d(a, b) = e \Leftrightarrow a = b$;
3. $d(a, b) = d(b, a)$;
4. $d(a, b) \leq d(a, c) \odot d(b, c)$.

$$d_{\mathcal{G}} : G \times G \to G$$
$$(a, b) \mapsto d_{\mathcal{G}}(a, b) = (a \div b) \vee (b \div a)$$

Real alo-groups

- An alo-group $\mathcal{G} = (G, \odot, \leq)$ is a *real* alo-group if and only if G is a subset of the real line $\mathbb{R}$ and $\leq$ is the total order on G inherited from the usual order on $\mathbb{R}$.
- If G is an interval of $\mathbb{R}$, then it has to be an open interval

Divisible alo-group

Definition

$\mathcal{G} = (G, \odot, \leq)$ is divisible if and only if $(G, \odot)$ is divisible, that is for each $n \in \mathbb{N}$ and each $a \in G$, the equation $x^{(n)} = a$ has at least a solution.

If $\mathcal{G} = (G, \odot, \leq)$ is divisible, then the equation $x^{(n)} = a$ has a unique solution. Thus, we give the following definition:

Definition

[3] Let $\mathcal{G} = (G, \odot, \leq)$ be divisible, $n \in \mathbb{N}$ and $a \in G$. Then, the *(n)-root* of a, denoted by $a^{(\frac{1}{n})}$, is the unique solution of the equation $x^{(n)} = a$, that is:

$$(a^{(\frac{1}{n})})^{(n)} = a.$$

- Two divisible continuous real alo-groups are isomorphic.

-mean

Definition

[3] Let $\mathcal{G} = (G, \odot, \leq)$ be divisible. $\odot$-*mean* $m_\odot(a_1, a_2, ..., a_n)$ of the n elements $a_1, a_2, ..., a_n$ of G is the element $a \in G$ verifying the equality $a \odot a \odot ... \odot a = a_1 \odot a_2 \odot ... \odot a_n$; that is

$$m_\odot(a_1, a_2, ..., a_n) = \begin{cases} a_1 & n = 1, \\ (\bigodot_{i=1}^{n} a_i)^{(\frac{1}{n})} & n \geq 2. \end{cases}$$

r-power

$$I_{a,r} = \{a^{(q)} : q \in Q \quad and \quad q < r\}$$

$$S_{a,r} = \{a^{(q)} : q \in Q \quad and \quad q > r\}$$

Definition

Let $\mathcal{G} = (G, \odot, \leq)$ be a real divisible continuous alo-group. For each $a \in G$ and $r \in \mathbb{R}$, $a^{(r)}$ is the separation point of $I_{a,r}$ and $S_{a,r}$, thus the following holds:

$$a^{(r)} = h((h^{-1}(a))^r),$$

with h an isomorphism between **]0,+∞[** and $\mathcal{G}$.

Example of real divisible alo-groups

Multiplicative alo-group. **]0,+∞[** = $(]0, +\infty[, \cdot, \leq)$, where $\cdot$ is the usual multiplication on $\mathbb{R}$

Additive alo-group. $\mathcal{R} = (\mathbb{R}, +, \leq)$, where $+$ is the usual addition on $\mathbb{R}$

Fuzzy group. **]0,1[**$=(]0, 1[, \otimes, \leq)$, where $\otimes :]0, 1[^2 \rightarrow]0, 1[$ is the operation defined by

$$x \otimes y = \frac{xy}{xy + (1 - x)(1 - y)}$$

Multiplicative alo-group

$$\mathbf{]0,+\infty[} = (]0,+\infty[, \cdot, \leq)$$

- $e = 1$
- $a^{(-1)} = 1/a$
- $a \div b = \frac{a}{b}$
- $a^{(r)} = a^r$
- $d_{\mathbf{]0,+\infty[}}(a,b) = \frac{a}{b} \vee \frac{b}{a}$
- $m.(a_1, ..., a_n) = \left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}}$

Additive alo-group

$\mathcal{R} = (\mathbb{R}, +, \leq)$

- $e = 0$
- $a^{(-1)} = -a$
- $a \div b = a - b$
- $a^{(r)} = r \cdot a$
- $d_{\mathcal{R}}(a, b) = |a - b| = (a - b) \vee (b - a)$
- $m_{+}(a_1, ..., a_n) = \frac{\sum_i a_i}{n}$
- $l : x \in]0, +\infty[\mapsto \log x \in \mathbb{R}$

Fuzzy alo-group

$$\mathbf{]0,1[} = (]0,1[, \otimes, \leq)$$

- $e = 0.5$
- $a^{(-1)} = 1 - a$
- $a \div b = \frac{a(1-b)}{a(1-b)+(1-a)b}$
- $a^{(r)} = \frac{a^r}{a^r+(1-a)^r}$
- $d_{\mathbf{]0,1[}}(a,b) = \frac{a(1-b)}{a(1-b)+(1-a)b} \vee \frac{b(1-a)}{b(1-a)+(1-b)a}$
- $m_{\otimes}(a_1,\dots a_n) = \frac{\sqrt[n]{\prod_{i=1}^{n} a_i}}{\sqrt[n]{\prod_{i=1}^{n} a_i} + \sqrt[n]{\prod_{i=1}^{n}(1-a_i)}}$
- $\psi : x \in]0,+\infty[\mapsto \frac{x}{x+1} \in]0,1[$

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PCMs on real divisible alo-groups

- $X = \{x_1, x_2, ...x_n\}$ is a set of alternatives
- $\mathcal{G} = (G, \odot, \leq)$ is a real continuous divisible alo-group
- $A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ defined on $\mathcal{G} \Leftrightarrow a_{ij} \in G$
- $\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n$ the rows of A
- $\underline{a}^1, \underline{a}^2, \dots, \underline{a}^n$ the columns of A
- $\underline{w}_{m_\odot}(A) = (m_\odot(\underline{a}_1), m_\odot(\underline{a}_2), \cdots, m_\odot(\underline{a}_n))$

PCMs on real divisible alo-groups

⊙- Reciprocity

$$a_{ji} = a_{ij}^{(-1)} \quad \forall i,j,k \in \{1,\ldots n\}$$

- $a_{ii} = e$
- $A = \begin{pmatrix} e & a_{12} & \ldots & a_{1n} \\ a_{12}^{(-1)} & e & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{1n}^{(-1)} & a_{2n}^{(-1)} & \ldots & e \end{pmatrix}$

Examples of reciprocal PCMs

Example

$$(]0,+\infty[,\cdot,\leq)$$

$$\begin{pmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{12}^{-1} & 1 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{1n}^{-1} & a_{2n}^{-1} & \dots & 1 \end{pmatrix}$$

$$a_{ji} = \frac{1}{a_{ij}}$$

Example

$$(\mathbb{R},+,\leq)$$

$$\begin{pmatrix} 0 & a_{12} & \dots & a_{1n} \\ -a_{12} & 0 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ -a_{1n} & -a_{2n} & \dots & 0 \end{pmatrix}$$

$$a_{ji} = -\ a_{ij}$$

Example

$$(]0,1[,\otimes,\leq)$$

$$\begin{pmatrix} 0.5 & a_{12} & \dots & a_{1n} \\ 1-a_{12} & 0.5 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 1-a_{1n} & 1-a_{2n} & \dots & 0.5 \end{pmatrix}$$

$$a_{ji} = 1-\ a_{ij}$$

PCMs on real divisible alo-groups

$\odot$- Consistency

$$a_{ik} = a_{ij} \odot a_{jk} \quad \forall i, j, k \in \{1, \ldots n\}$$

- $CM(n)$ is the set of $\odot$-consistent PCMs of order n
- Consistency shortcoming is removed

Examples of consistent PCMs

Example

$$(]0,+\infty[,\cdot,\leq) \qquad A=\begin{pmatrix} 1 & 2 & 6 \\ \frac{1}{2} & 1 & 3 \\ \frac{1}{6} & \frac{1}{3} & 1 \end{pmatrix}$$

$$a_{ik} = a_{ij}\ a_{jk}$$

Example

$$(\mathbb{R},+,\leq) \qquad A=\begin{pmatrix} 0 & 2 & 5 \\ -2 & 0 & 3 \\ -5 & -3 & 0 \end{pmatrix}$$

$$a_{ik} = a_{ij} + a_{jk}$$

Example

$$(]0,1[,\otimes,\leq) \qquad A=\begin{pmatrix} 0.5 & 0.6 & 0.\overline{7} \\ 0.4 & 0.5 & 0.7 \\ 1-0.\overline{7} & 0.3 & 0.5 \end{pmatrix}$$

$$a_{ik} = \frac{a_{ij}a_{jk}}{a_{ij}a_{jk} + \left(1-a_{ij}\right)\left(1-a_{jk}\right)}$$

PCMs on real divisible alo-groups

Definition

Let $A = (a_{ij}) \in CM(n)$. A vector $\underline{w} = (w_1, \ldots, w_n)$, with $w_i \in G$, is a $\odot$-*consistent* vector for $A = (a_{ij})$ if and only if:

$$w_i \div w_j = a_{ij} \quad \forall\, i, j \in \{1, \ldots, n\}$$

Proposition

The following assertions related to $A = (a_{ij})$ are equivalent:

1. $A = (a_{ij}) \in CM(n)$
2. *there exists a $\odot$-consistent vector $\underline{w}$ for A*
3. *each column $\underline{a}^k$ is a $\odot$-consistent vector*
4. *the $\odot$-mean vector $\underline{w}_{m_\odot}(A)$ is a $\odot$-consistent vector*

PCMs on real divisible alo-groups

Definition

Let $A \in RM(n)$ with $n \geq 3$, then the $\odot$-consistency index $I_{\mathcal{G}}(A)$ is defined as follows:

$$I_{\mathcal{G}}(A) = \Big(\bigodot_{i<j<k} d_{\mathcal{G}}(a_{ik}, a_{ij} \odot a_{jk}) \Big)^{\left(\frac{1}{n_T}\right)}$$

with $n_T = \frac{n(n-2)(n-1)}{6}$.

$$I_{\mathcal{G}}(A) = \begin{cases} d_{\mathcal{G}}(a_{13}, a_{12} \odot a_{23}) & n = 3, \\ \left(\bigodot_{i<j<k} I_{\mathcal{G}}(A_{ijk})\right)^{\left(\frac{1}{n_T}\right)} & n > 3. \end{cases}$$

- $I_{\mathcal{G}}(A)$ has an intuitive meaning, because is a $\odot$-mean of $\mathcal{G}$-distances;
- $I_{\mathcal{G}}(A)$ is suitable for several kinds of pairwise comparisons matrices (e.g. multiplicative, additive and fuzzy).

Example - multiplicative

Example

$$A = \begin{pmatrix} 1 & \frac{1}{7} & \frac{1}{7} & \frac{1}{5} \\ 7 & 1 & \frac{1}{2} & \frac{1}{3} \\ 7 & 2 & 1 & \frac{1}{9} \\ 5 & 3 & 9 & 1 \end{pmatrix}$$

$$I_{]0,+\infty[}(A) =$$
$$\sqrt[4]{I_{]0,+\infty[}(A_{123}) \cdot I_{]0,+\infty[}(A_{124}) \cdot I_{]0,+\infty[}(A_{134}) \cdot I_{]0,+\infty[}(A_{234})} =$$
$$\sqrt[4]{2 \cdot \frac{21}{5} \cdot \frac{63}{5} \cdot 6} = 5.02$$

Example - additive

Example

$$B = \begin{pmatrix} 0 & -\log 7 & -\log 7 & -\log 5 \\ \log 7 & 0 & -\log 2 & -\log 3 \\ \log 7 & \log 2 & 0 & -\log 9 \\ \log 5 & \log 3 & \log 9 & 0 \end{pmatrix}$$

$$I_{\mathcal{R}}(B) = \frac{I_{\mathcal{R}}(B_{123}) + I_{\mathcal{R}}(B_{124}) + I_{\mathcal{R}}(B_{134}) + I_{\mathcal{R}}(B_{234})}{4} = \frac{0.6931 + 1.4350 + 2.5336 + 1.7917}{4} = 1.6134$$

$$I_{\mathcal{R}}(B) = \log(I_{]0,+\infty[}(A)) = \log(5.02) = 1.6134$$

Example - fuzzy

Example

$$C = \begin{pmatrix} 0.5 & 0.3 & 0.4 & 0.4 \\ 0.7 & 0.5 & 0.1 & 0.2 \\ 0.6 & 0.9 & 0.5 & 0.8 \\ 0.6 & 0.8 & 0.2 & 0.5 \end{pmatrix}$$

$$I_{]0,1[}(C) = \frac{\sqrt[4]{\prod_{i<j<k} I_{]0,1[}(C_{ijk})}}{\sqrt[4]{\prod_{i<j<k} I_{]0,1[}(C_{ijk})} + \sqrt[4]{\prod_{i<j<k}(1 - I_{]0,1[}(C_{ijk}))}} = 0.833$$

$$I_{]0,1[}(C) = \psi(I_{]0,+\infty[}(C')) = \frac{4.9888}{5.9888} = 0.833.$$

Consistency index and mean vector

$$A=\begin{pmatrix} e & a_{12} & \dots & a_{1n} \\ a_{12}^{(-1)} & e & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{1n}^{(-1)} & a_{2n}^{(-1)} & \dots & e \end{pmatrix} \rightarrow w_{m_{\odot}} = (w_1, \dots, w_n), \quad w_i = m_{\odot}(a_{i1}, \dots, a_{in})$$

A consistent $\leftrightarrow$ $w_{m_{\odot}}$ consistent vector

$$d_G(a_{ij}, w_i \div w_j) \le I_G(A)^{\left(\frac{(n-2)(n-1)}{6}\right)}$$

The more $I_G(A)$ is close to e
the more A is close to be a consistent PCM

Consistency index

- *Let* $(G,\odot,\leq)$ *be a* divisible ALO group.
- $I_G(A) \geq e$
- $I_G(A)=e \Leftrightarrow A$ *is consistent*
  - $I_G(A)=1 \Leftrightarrow A$ *is consistent (multiplicative case)*
  - $I_G(A)=0 \Leftrightarrow A$ *is consistent (additive case)*
  - $I_G(A)=0.5 \Leftrightarrow A$ *is consistent (fuzzy case)*

Different levels of coherence

- Transitivity

$$a_{ij} \geq e,\ a_{jk} \geq e \Rightarrow a_{ik} \geq e.$$

- Weak consistency

$$a_{ij} \geq e \quad a_{jk} \geq e \Rightarrow a_{ik} \begin{cases} = max\{a_{ij}, a_{jk}\} & if\ a_{ij} = e\ or\ a_{jk} = e, \\ > max\{a_{ij}, a_{jk}\} & otherwise. \end{cases}$$

- Consistency

$$a_{ik} = a_{ij} \odot a_{jk} \quad \forall i, j, k = 1, \ldots, n.$$

Future work

- General structures encompassing several cases of consistency and coherence principles of decision making should be investigated
- Tentative candidates: MV-algebras

Definition An *MV-algebra* $\langle A, \oplus, \neg, 0\rangle$ is a set A equipped with a binary operation $\oplus$, a unary operation $\neg$ and a distinguished constant 0 satisfying the following equations:

(1.1) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$

(1.2) $x \oplus y = y \oplus x$

(1.3) $x \oplus 0 = x$

(1.4) $\neg\neg x = x$

(1.5) $x \oplus \neg 0 = \neg 0$

(1.6) $\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x.$

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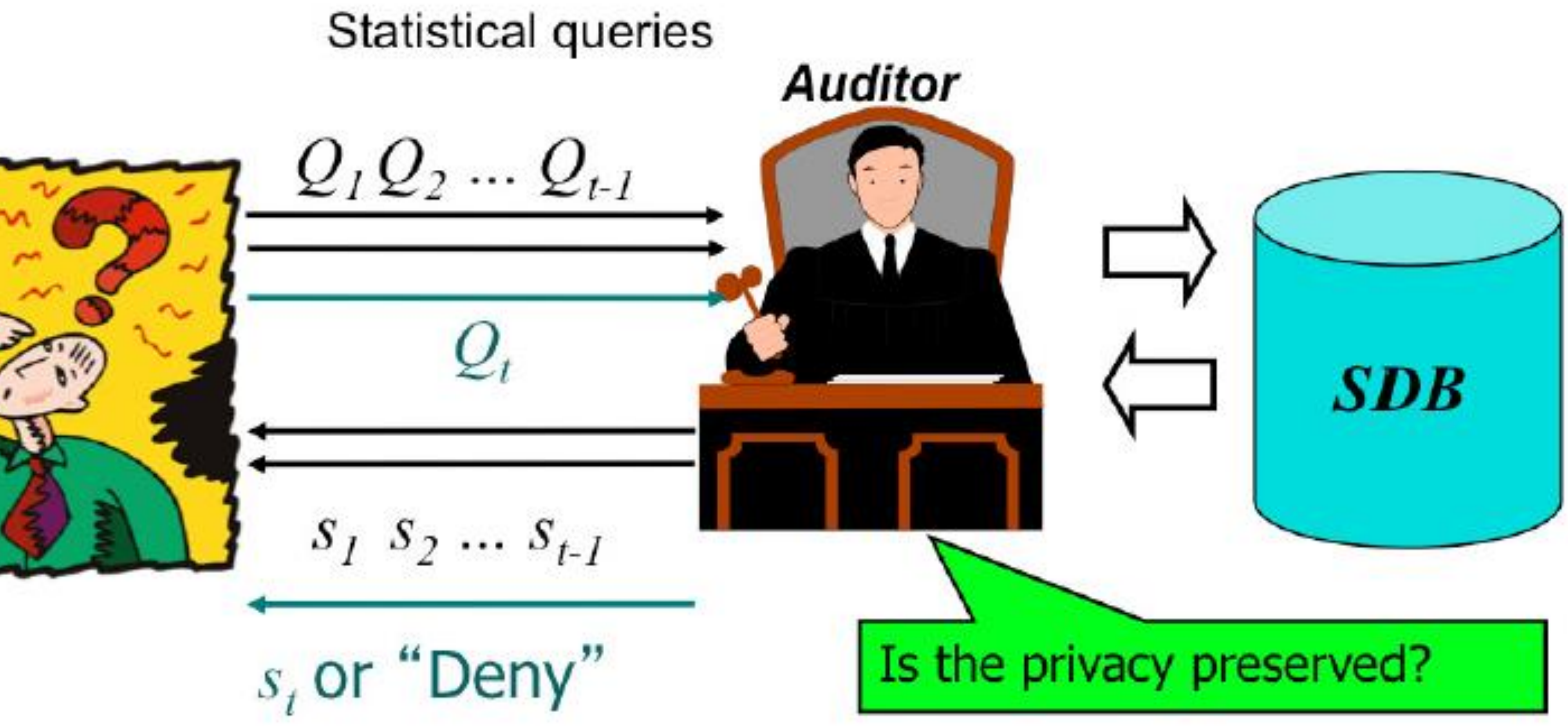
  - Management of human resources

The approach

- It encompasses:
  - disclosure risk assessment in Statistical Databases
  - privacy requirements prioritization
- In particular:
  - Bayesian networks for on-line auditing in Statistical Databases
  - Pairwise Comparison Matrices for privacy requirements prioritization

On-line auditing in Statistical databases



ıte M.

Example of on-line max Auditing in Statistical databases on a real domain

max Salary{Alice,Bob,Carl} →

← 80

max Salary{Alice,Bob} →

← **denied**

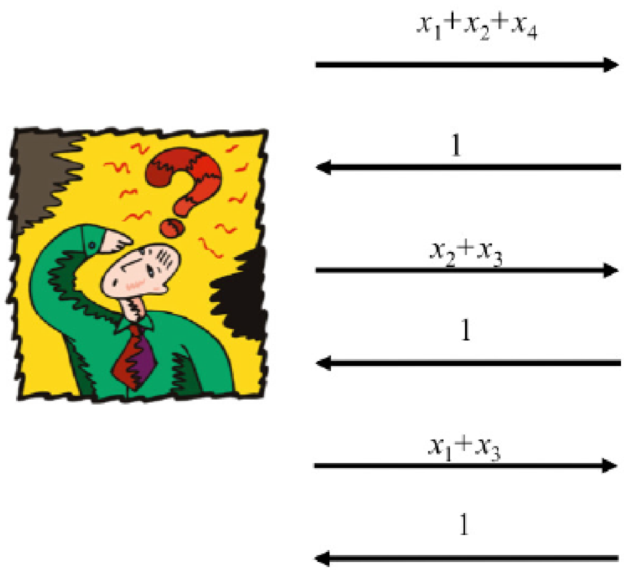
Company Database

Name	***Age***	***Sex***	***Salary***
Alice	*23*	*F*	*42*
Bob	*25*	*M*	*50*
Carl	*30*	*M*	*80*
Dave	*21*	*M*	*35*

On-line Sum/Count/Mean/Max/Min Auditing on a Boolean domain

- Example of sum auditing

Let X be the sensitive field.



X	*Name*	*Age*	*Sex*
x_1=?	Alice	34	F
x_2=?	Bob	35	M
x_3=?	Carl	40	M
x_4=?	Dave	27	M

$$\begin{cases} x_1 + x_2 + x_4 = 1 \\ x_2 + x_3 = 1 \\ x_1 + x_3 = 1 \end{cases} \longrightarrow (x_1, x_2, x_3, x_4) = (0, 0, 1, 1).$$

$$\Downarrow$$

Privacy is breached.

Probabilistic compromise

Definition

[3] A privacy breach occurs if and only if a sensitive data is disclosed with probability greater or equal to a given tolerance probability *tol*.

If a sensitive data is disclosed with $tol = 1$, then the SDB is fully compromised.

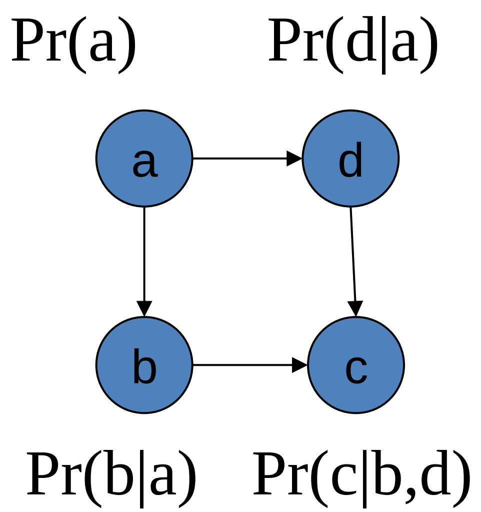
On-line auditing by means of a Bayesian Network

- A Bayesian network (BN) is a probabilistic graphical model that represents a set of variables and their probabilistic dependencies
  - is a directed acyclic graph (DAG)
    - nodes represent variables
    - arcs represent dependencies between variables
    - if there is an arc from node A to another node B
      - A is called a parent of B
      - B is a child of A
    - the strength of an influence between variables is represented by conditional probability table (CPT)

$$size(CPT) = s\prod_{j=1}^{n} s_j$$

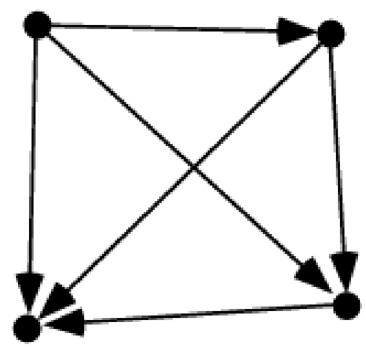
- Bayes' formula:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$



Graphs

- An *undirected graph G* is a pair *(V,E)*, where *V* is a finite set of points called *vertices* and *E* is a finite set of *arcs*
- An arc $e \in E$ is an unordered pair (u,v), where $u,v \in V$
- In a directed graph, the arc e is an ordered pair (u,v)
- A directed acyclic graph is a direct graph without cycles



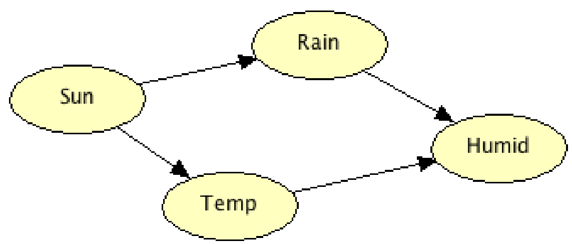
- A *path* from a vertex *v* to a vertex *u* is a sequence $<v_0,v_1,v_2,\ldots,v_k>$ of vertices where $v_0 = v$, $v_k = u$, and $(v_i, v_{i+1}) \in E$ for $i=0,1, \ldots,k-1$
- The length of a path is defined as the number of edges in the path

Example of BN

HUGINEXPERT
The leading decision support tool

Rain | Sun | Temp | Humid

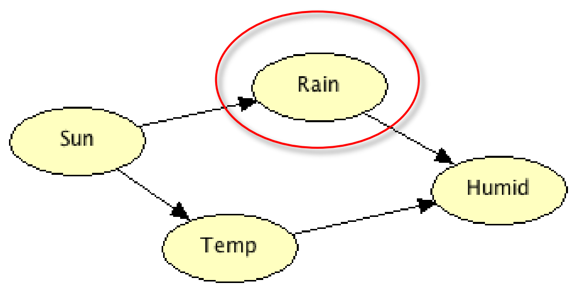
false	0.7
true	0.3



Example of BN

Rain | Sun | Temp | Humid

Sun	false	true
false	0.4	1
true	0.6	0



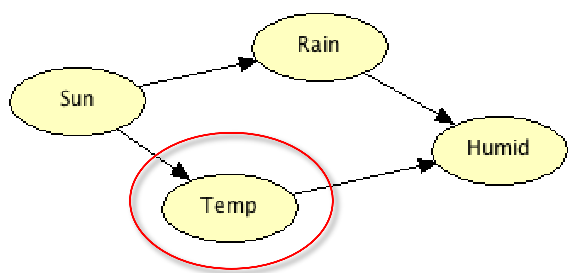
CPT(size)=states_number(Rain) x states_number (Sun)=2 x 2=4

Example of BN

HUGINEXPERT
The leading decision support tool

Rain | Sun | Temp | Humid

Sun	false	true
low	0.5	0.2
high	0.5	0.8



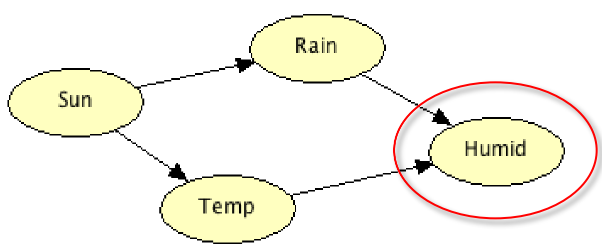
CPT(size)=states_number(Temp) x states_number (Sun)=2 x 2=4

Example of BN

HUGINEXPERT
The leading decision support tool

Rain | Sun | Temp | Humid

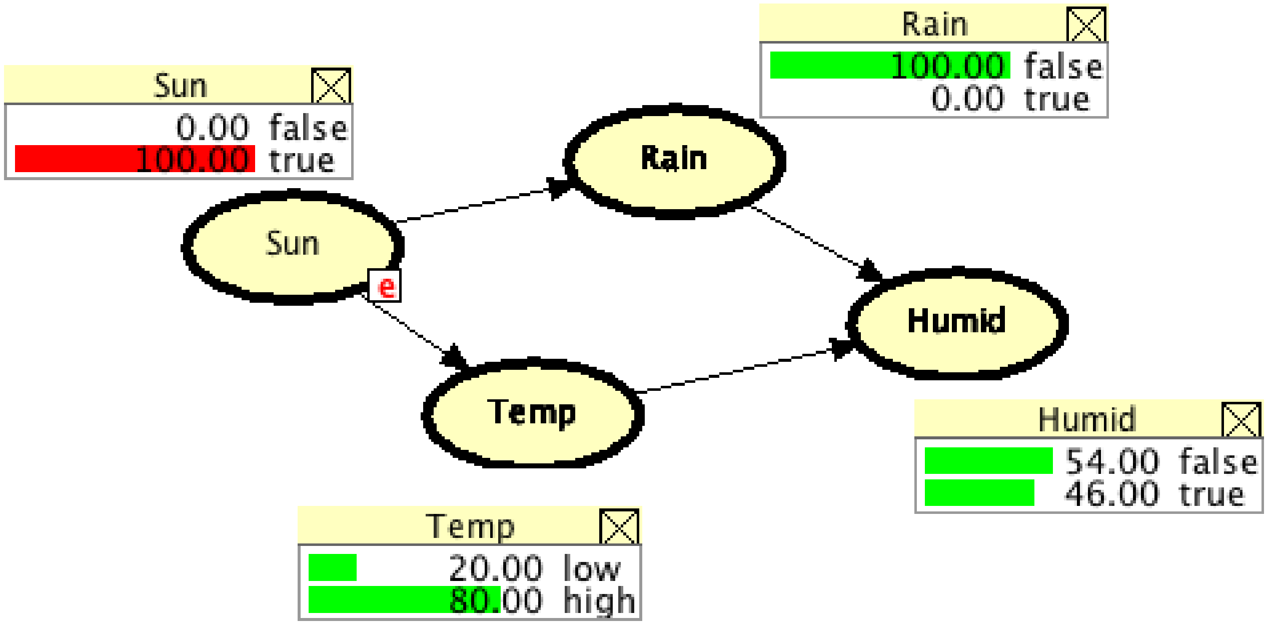
Temp	low		high	
Rain	false	true	false	true
false	0.7	0.6	0.5	0.1
true	0.3	0.4	0.5	0.9



CPT(size)=states_number(Humid) x states_number(Rain) x states_number (Temp)= 2 x 2 x 2=8

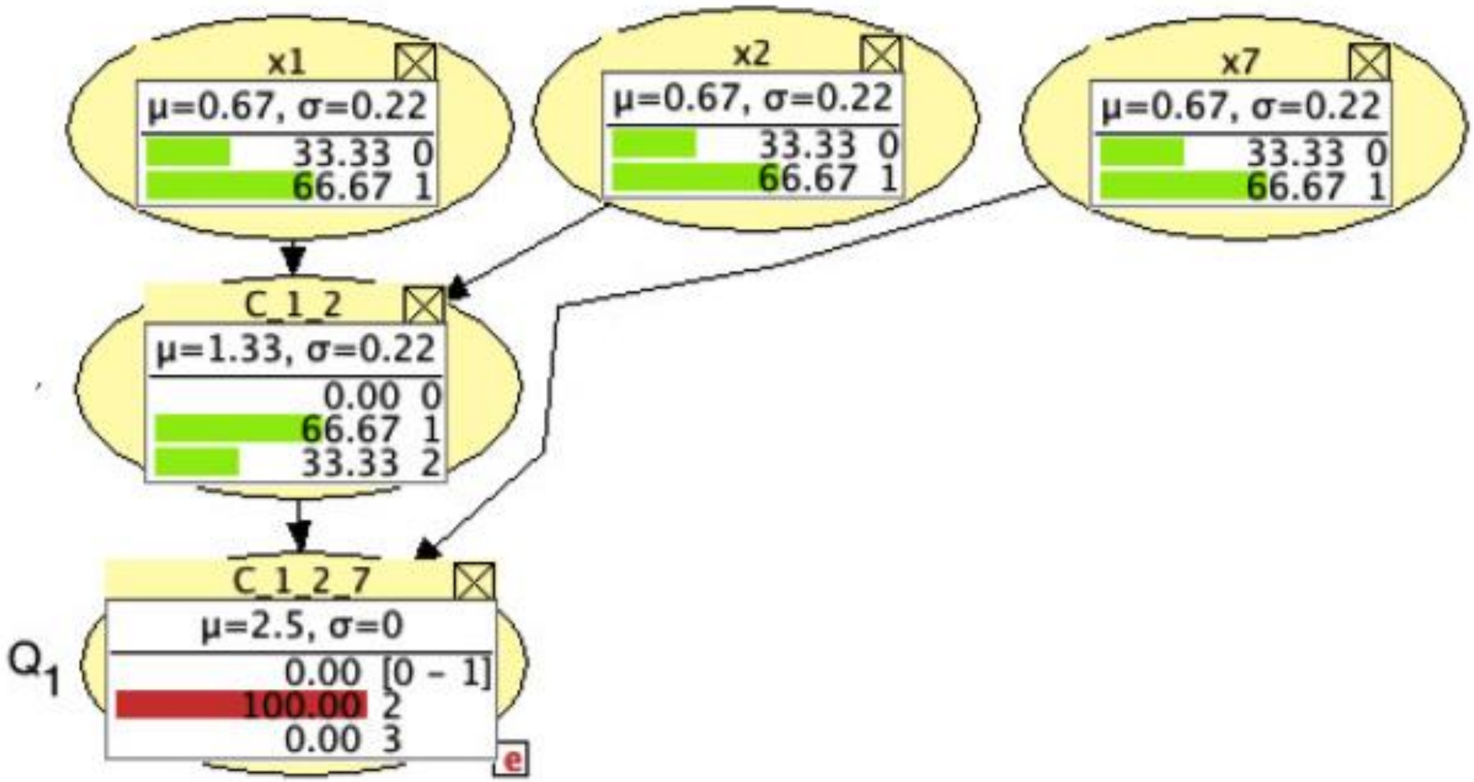
Example of BN

- Evidence on “Sun=true”



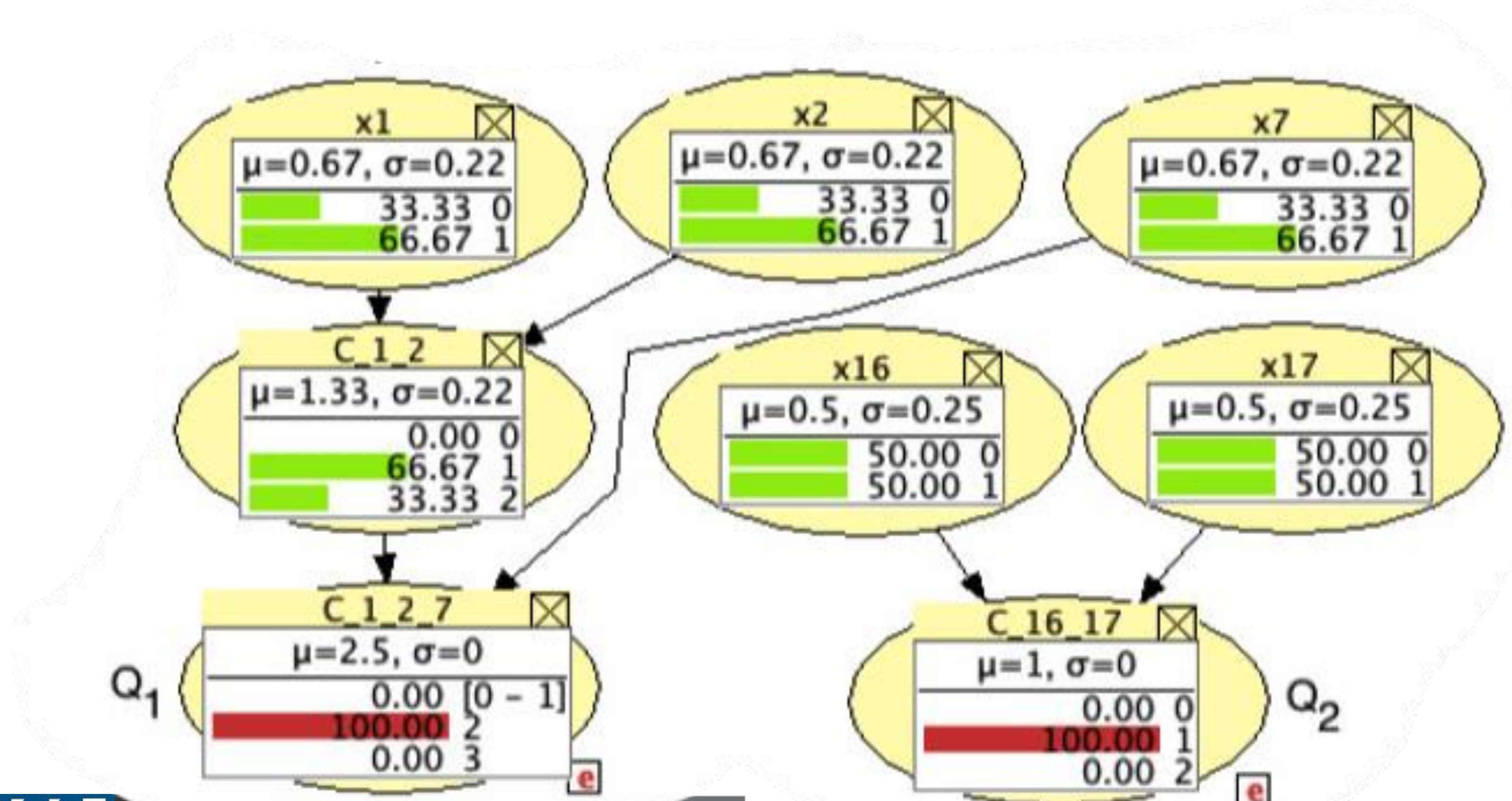
Example of a sequence of sum/count queries on a boolean domain

- Tol=0.8
- $Q_1=\{x_1, x_2, x_7\}$



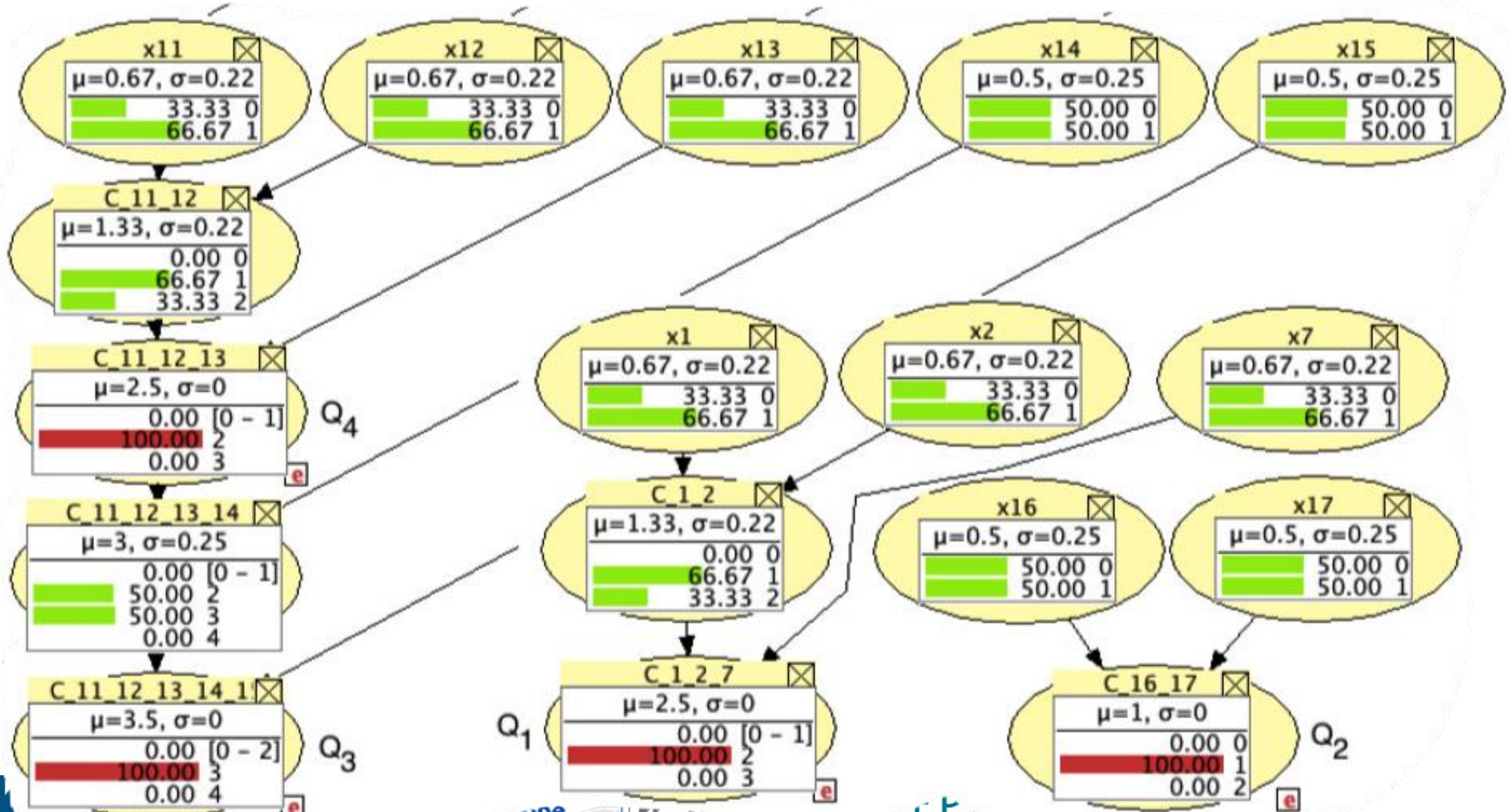
Example of a sequence of sum/count queries on a boolean domain

- Tol=0.8
- $Q_1=\{x_1, x_2, x_7\}$, $Q_2=\{x_{16}, x_{17}\}$



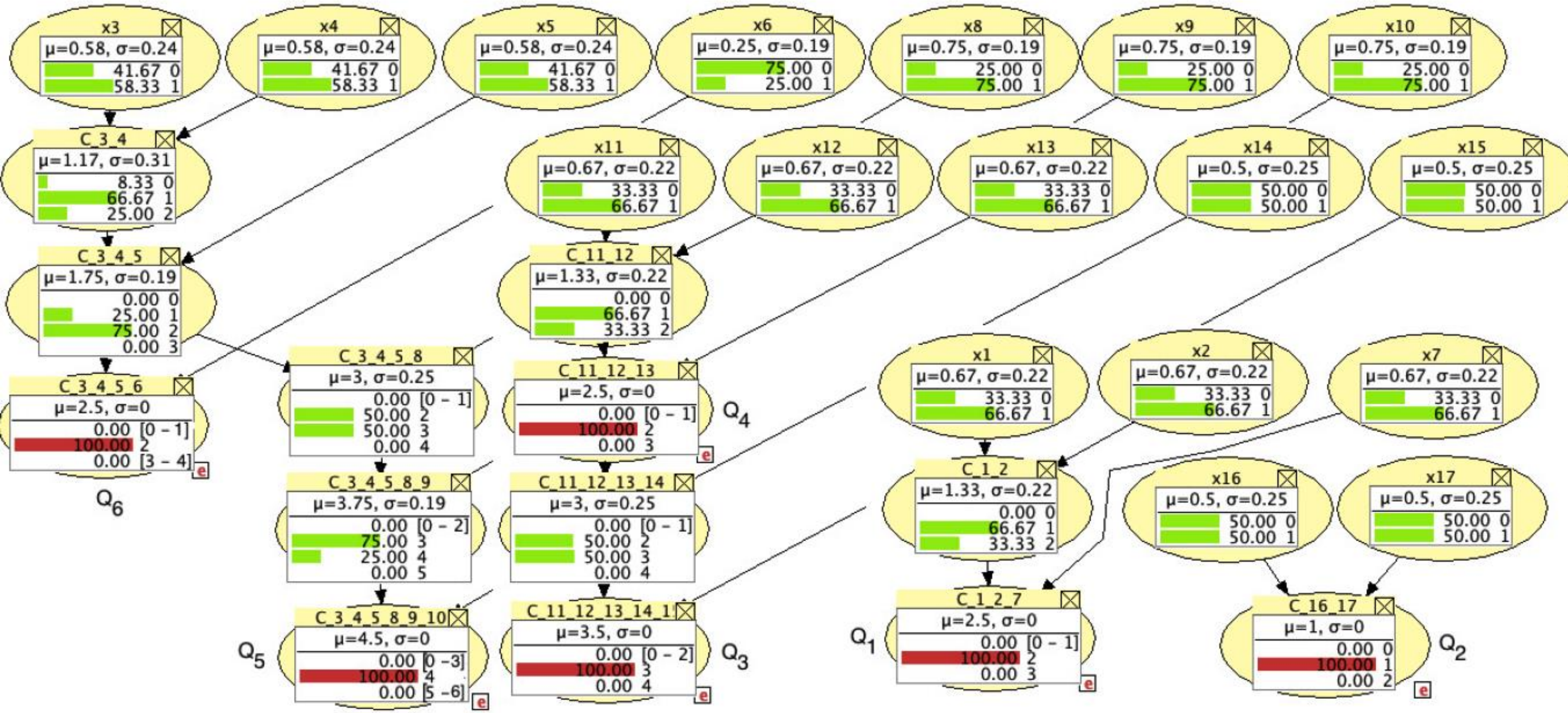
Example of a sequence of sum/count queries on a boolean domain

- Tol=0.8
- $Q_1=\{x_1, x_2, x_7\}$, $Q_2=\{x_{16}, x_{17}\}$, $Q_3=\{x_{11}, x_{12}, x_{13}, x_{14}, x_{15}\}$, $Q_4=\{x_{11}, x_{12}, x_{13}\}$,



Example of a sequence of sum/count queries on a boolean domain

- Tol=0.8
- $Q_1=\{x_1, x_2, x_7\}$, $Q_2=\{x_{16}, x_{17}\}$, $Q_3=\{x_{11}, x_{12}, x_{13}, x_{14}, x_{15}\}$, $Q_4=\{x_{11}, x_{12}, x_{13}\}$, $Q_5=\{x_3, x_4, x_5, x_8, x_9, x_{10}\}$, $Q_6=\{x_3, x_4, x_5, x_6\}$



Remark

- The above BN has been proposed by Cavallo B. and Canfora G. (2015)
- It optimizes the model previously proposed by Canfora and Cavallo (2010):
  - by reducing the CPT size of the BN for auditing a query of size l, from $O(2^l)$ to $O(l^3)$, by means of a temporal transformation (this works whenever an associative operator has to be implemented)
  - by reducing the CPT size at run-time, given the answer to the current query (i.e. by unifying the states with probability equal to 0)

Privacy requirements prioritization

- Data privacy plays a central role in many modern applications
  - it is needed to plan and design a system with privacy in mind.

- Software functional requirements are analysed up-front to make sure that the planned system will meet user needs in terms of services being delivered
- Privacy requirements must be defined up-front to satisfy the needs of the customers and to comply with laws, standards and service policies.

Example: e-banking service

- Let us consider a bank that has to develop an e-banking service for allowing his customers to conduct financial transactions on a secure website.
- The bank will be directly responsible for the safety and privacy of the e-banking service.
- The system will have to satisfy privacy requirements in addition to functional and security ones.

Example: e-banking service

1. clearly articulate the level of customer privacy and at what extent his/her information (e.g. salary, credit cards number, electronic funds transfer) will be exposed internally within the bank (e.g., operator, database administrator, manager);
2. a customer must be given the option of not giving their personally identifiable information if the information collected is not related to the primary purpose for which the information was collected;
3. the customers choice about personally identifiable information being disclosed to third parties must be honoured. The customer must also have the means to change their choice;
4. provide information about how personally identifiable information are collected by the site.

Example: e-banking service

- Let us suppose that a bank's DM expresses his/her preference intensities

$$A = \begin{pmatrix} 1 & 2 & 4 & 5 \\ \frac{1}{2} & 1 & 2 & \frac{5}{2} \\ \frac{1}{4} & \frac{1}{2} & 1 & \frac{5}{4} \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} & 1 \end{pmatrix}$$

$$\underline{w}_m(A) = \left(\sqrt[4]{40}, \sqrt[4]{\frac{5}{2}}, \sqrt[4]{\frac{5}{32}}, \sqrt[4]{\frac{8}{125}} \right) \longrightarrow x_1 \succ x_2 \succ x_3 \succ x_4,$$

Awards

- **TR35 young innovators**
  - Organized by MIT Technology Review (Italia) and Forum Ricerca Innovazione Imprenditorialità.
    - TR35-GI: Protezione della privacy dei dati mediante reti bayesiane. Technol. Rev. **2,** 10 (2011)
- **DYSES 2010-** Best paper Award for the paper.
  - Bice Cavallo, Livia D'Apuzzo, Massimo Squillante. About a consistency index for pairwise comparison matrices over a divisible alo-group. Int. J. Intell. Syst. 27(2): 153-175 (2012)

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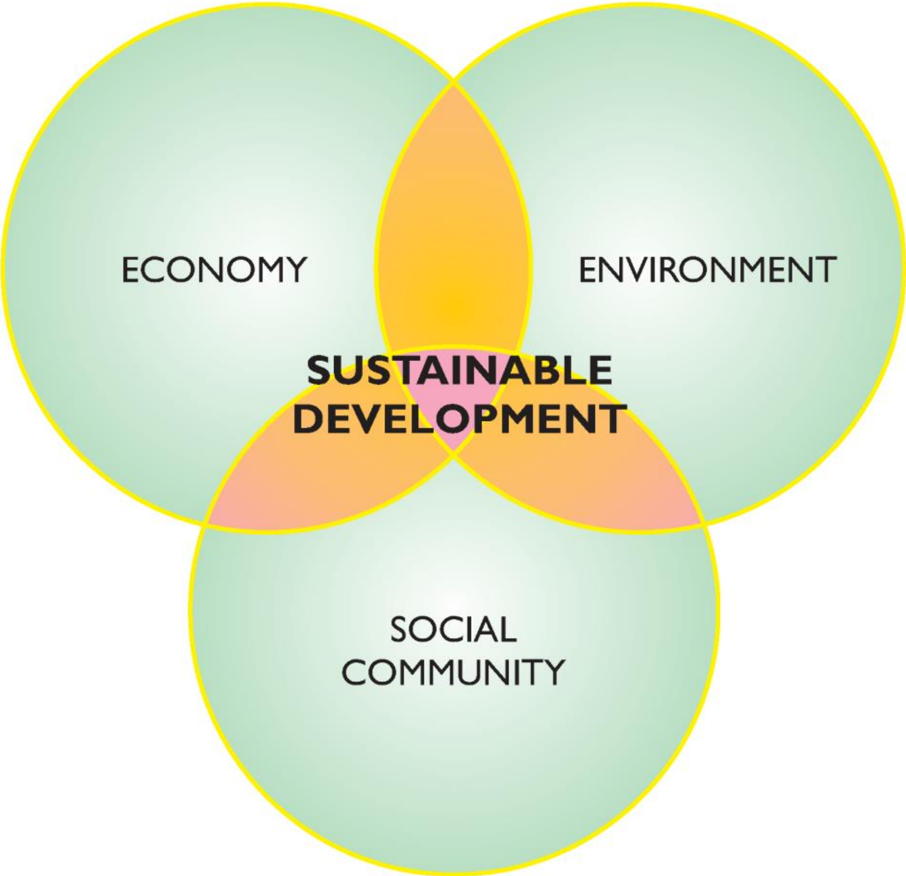
Sustainable development

- The territory is the result of a multiplicity of components (e.g. economic, environment, social) that interact among them.
- One cannot ignore this multidimensional nature in evaluation and choice of actions, and needs a careful understanding of the effects, not only in the context of reference, but also on other areas of the urban system where these effects can occur.



The application

- We propose an application of the Analytic Hierarchy Process for the sustainable urban development of Naples port area, by focusing on economic, environment and social impact.
- Then, we perform a preliminary experimentation, by means of an opinion survey with a sample of students of Department of Architecture and Department of Economic, Legal and Social Studies.
- In particular, we get a ranking of the alternatives that highlights that the preferable alternative aims to improve services for tourism and leisure, to enjoy the sea and the monumental area.

Development of a hierarchycal decision model

- **O** Sustainable urban development of Naples port area.
  - **O1** Economic sustainability: pursuing the economic needs of the port area;
  - **O2** Environment sustainability:maintaining quality and reproducibility of natural resources;
  - **O3** Social sustainability: ensuring conditions of human well-being (safety, health, education) evenly distributed by class and gender.

Development of a hierarchycal decision model

Relative to the sub-objective **O1**

- **C11** Optimizing infrastructures, particularly by improving the road network (e.g. road trim, flow of traffic, links to the nearby beach areas and to monumental interest areas);
- **C12** Creating new employment in the port area (e.g. encouraging the development of business activities, sales of local products, building a shopping center);
- **C13** Involving private operators (e.g. coordination and integration of ship-owners, merchants, service providers);
- **C14** Promoting interventions that raise the tourism-related activities (e.g. increasing the attractiveness of the port area and surroundings during all seasons, providing guides and tourist routes);
- **C15** Creating innovative services for citizens and tourists (e.g.multimedia fruition of tourism services, museum, information, online reservations of hotels and transport);
- **C16** Redevelopment of the built heritage of the port and monumental area.

Development of a hierarchycal decision model

Relative to the sub-objective **O2**,

- **C21** Creating a small green oasis, flower beds, hedges, and planting typical Mediterranean plants;
- **C22** Improving water quality and cleaning up the seabed;
- **C23** Implementing sustainable mobility between the port and the surrounding residential areas, in order to reduce the high level of air and noise pollution;
- **C24** Using renewable energy in building interventions (e.g. solar panels, renewable materials).

Development of a hierarchycal decision model

Relative to the sub-objective **O3**

- **C31** Improving the quality of employment (e.g. adopting safety standards in the workplace, promoting technological innovation of machinery and equipment);
- **C32** Lowering the crime rate in the port area through the revitalization of the port of Naples and the establishment of safeguards surveillance;
- **C33** Improving quality of life, with a stronger focus on children, social integration and problems of marginalization (e.g. areas of children's entertainment, baby parking for workers, awareness-raising events);
- **C34** Strengthening human capital (e.g. education and research on activities of the sea, with particular reference to international law, yacht design and new materials for shipbuilding, marine biology).

Development of a hierarchycal decision model

- **A1** A scenario of facilities and cultural services (e.g, workshops, exhibition spaces, recovery and restoration of relics and artefacts of naval archaeology).
- It will be appropriate to involve universities (i.e. Federico II, Orientale, Sun, Suor Orsola Benincasa, Parthenope), in the management process of the UNESCO site, in order to enhance both the availability of the cultural heritage and the organization of intangible assets that allowthe strengthening of the vision of theUNESCOsite as City of widespread
- training.
- The scenario proposes a coordination between the various universities and connection between the world of research/universities and businesses, and between businesses and associations, both in terms of information disclosure and publication of the results.
- Finally, the scenario considers essential the strengthening of the system of reception and student services.

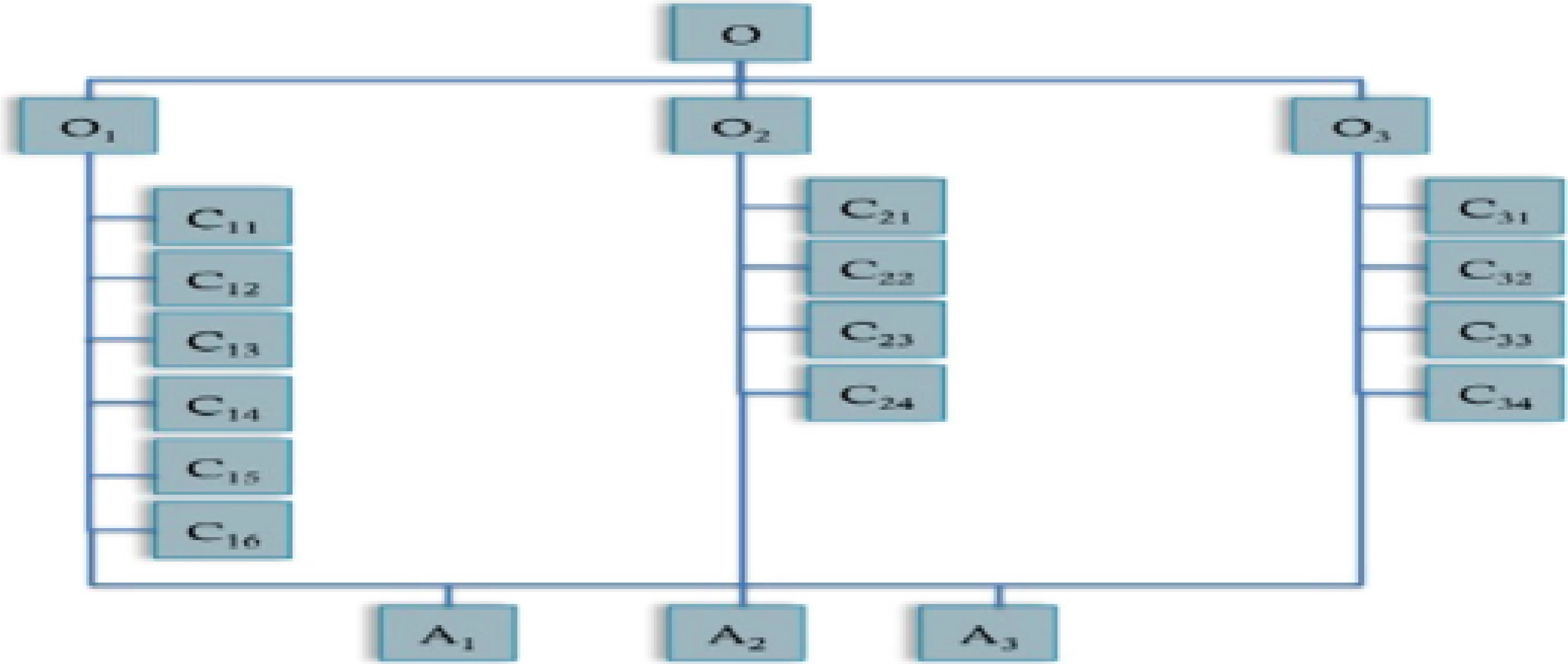
Development of a hierarchycal decision model

- **A2** A scenario of services for tourism and leisure activities, to enjoy the sea and the monumental area, and accommodate events (e.g. the Americas Cup World Series).
- By its nature of a "transit port", the port of Naples must be able to intercept and manage tourist flows directed towards other destinations, by arousing interest in Naples and, more generally, to the reality of the UNESCO sites of Campania.
- The scenario proposes a collaboration with tour operators in order to offer tour packages both directly on board cruise ships (one million three hundred thousand visitors per year) and by providing organized assistance in land for tours and excursions in historic centre, in Neapolitan hinterland, on the smaller islands and the sites bordering the Bay of Naples.
- As problems of crime undermine the tourist image of the city of Naples and endanger the safety of tourists, this scenario envisages the establishment of safeguards actions (e.g. safeguards surveillance, street lighting, video cameras) in the port area.
- Finally, this scenario proposes the development of medium-low hotel accommodation in order to improve the tourism flows, in addition to existing good availability of medium-high hotel accommodation.

Development of a hierarchycal decision model

- **A3** A scenario with a fair-trade center in the port area, to support producers, awareness raising and campaigning for changes in the rules and practice of conventional international trade.
- Naples boasts the presence of a first-order goldsmith pole in the heart of downtown (Borgo Orefici) and the presence of deep-rooted historical craft traditions, such as: the pizza, pastry craft, nativity scenes in S. Gregorio Armeno, production of bridal and communion gowns, holy dresses, restoration and sale of sacred objects (mainly in via Duomo).
- This scenario encourages a fair-trade center with the sales of local products and local handicraft, and the aggregation and integration to create groups, supply chains of production, forms of coordination between business enterprises.

The hierarchycal model



Evaluation matrix of criteria with respect to sub-objectives

	O_1 ($p_1 = 0.37$)	O_2 ($p_2 = 0.25$)	O_3 ($p_3 = 0.36$)
C_{11}	0,12		
C_{12}	0,23		
C_{13}	0,08		
C_{14}	0,21		
C_{15}	0,18		
C_{16}	0,17		
C_{21}		0,17	
C_{22}		0,30	
C_{23}		0,20	
C_{24}		0,33	
C_{31}			0,23
C_{32}			0,27
C_{33}			0,32
C_{34}			0,18

Evaluation matrix of alternatives with respect to criteria

	C_{11} 0.05	C_{12} 0.09	C_{13} 0.03	C_{14} 0.08	C_{15} 0.07	C_{16} 0.07	C_{21} 0.04	C_{22} 0.08	C_{23} 0.05	C_{24} 0.08	C_{31} 0.08	C_{32} 0.10	C_{33} 0.12	C_{34} 0.06
A_1	0.14	0.14	0.2	0.29	0.62	0.09	0.29	0.29	0.25	0.44	0.14	0.14	0.45	0.4
A_2	0.57	0.29	0.4	0.57	0.31	0.82	0.57	0.57	0.5	0.44	0.29	0.57	0.45	0.2
A_3	0.29	0.57	0.4	0.14	0.07	0.09	0.14	0.14	0.25	0.12	0.57	0.29	0.1	0.4

Global evaluations

- By applying a weighted arithmetic mean, we compute the global evaluations of alternatives with respect to the goal:

$$v(A_1) = 0.28, \quad v(A_2) = 0.47, \quad v(A_3) = 0.25.$$

Sensitivity analysis

In order to study how the output of our mathematical model can be apportioned to uncertainty in its inputs, we perform a sensitivity analysis. We stress that the main difference between the groups of students is related to the weights p_1, p_2 and p_3 of the sub-objectives; in particular we obtain:

$$p_1 = 0.31, \quad p_2 = 0.32, \quad p_3 = 0.37,$$

and

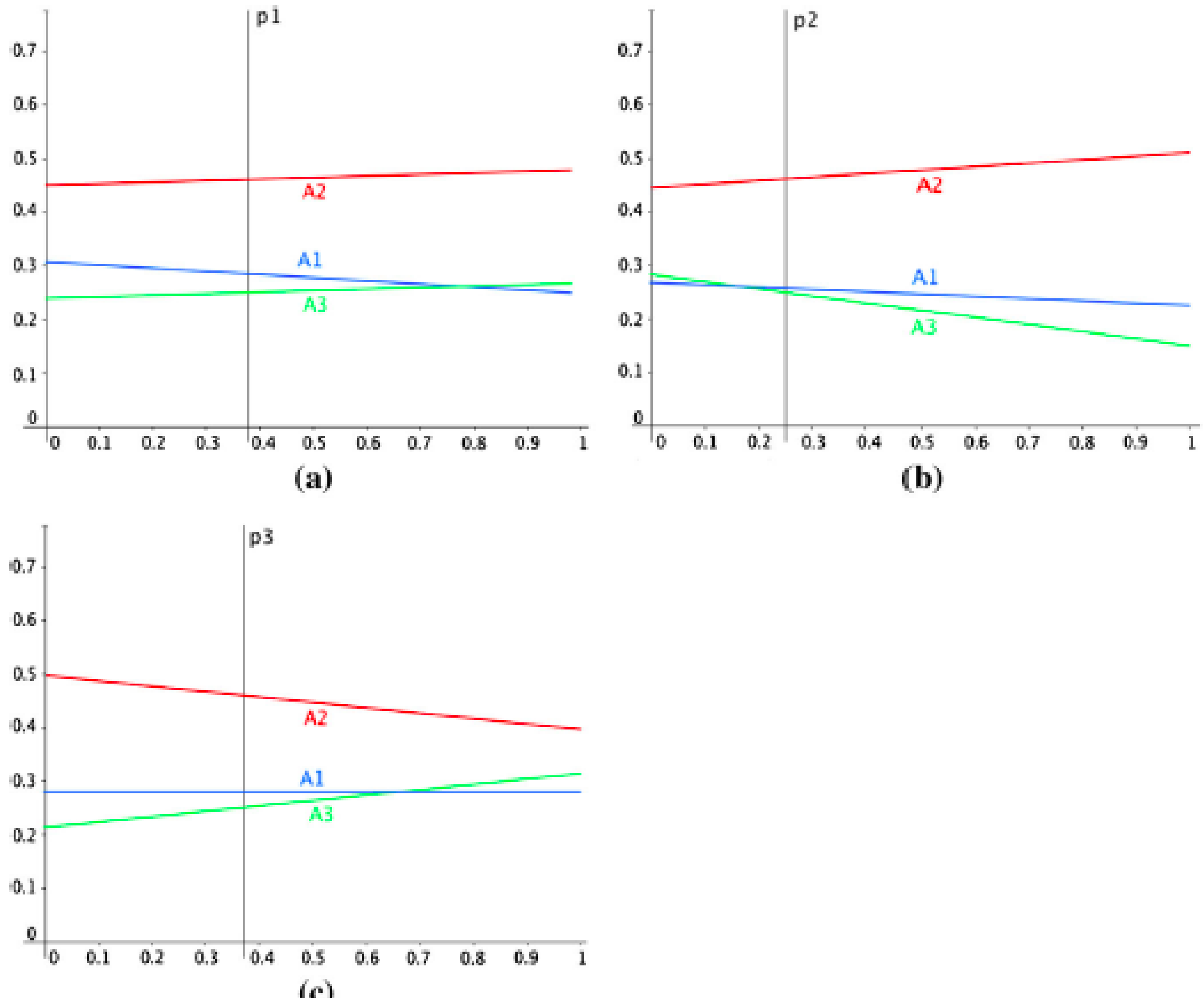
$$p_1 = 0.45, \quad p_2 = 0.19, \quad p_3 = 0.36,$$

by students of Department of Architecture and Department of Economic, Legal and Social Studies, respectively. The second sample of students prefers economic sustainability, that is pursuing the economic needs of the port area.

For this reason, we analyse how the weights p_1, p_2 and p_3 of the sub-objectives affect the choice of the alternative.

Sensitivity analysis

- The preferable alternative is always A2, contrariwise, there is sometimes an inversion of the preferences between A1 and A3.



Future work

- To consider a wider sample of stakeholders, such as institutional actors (e.g. Municipality of Naples, Campania Region, Port Authority, ASL), enterprises (e.g. experienced urban design practitioners, operators in the field of hospitality and catering), social and cultural actors (e.g.upper-school institutes, research institutes and universities, tourist promotion institutes, cultural associations, non-profit organizations, citizens, shipowners, workers, employees and merchant of Port of Naples)
- To investigate the inclusion of more comprehensive and dependent criteria by applying the analytic network process

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DESCRIPTION OF THE PROJECT

"Eco-substainable Requalification of existing buildings for Innovative tourist accommodation"

Reeti
Riqualificazione ecosostenibile edifici turismo innovativo

How it started:

by the continual increase of the tourist offer in Emilia Romagna and by the greater sensitization of local government in terms of energy and environmental sustainability.

The Objective:

REETI focuses on improving the accommodation facilities, offering requalification with energetic efficiency and reducing consumption of energy by using renewable sources, reducing environmental impact, improving the indoor comfort and the safety of buildings for tourist accommodation. (Project)

The Target:

Managers of tourist accommodation facilities, who wish to have a structure that will lead to reduced consumption, improved comfort and operation.

The Promoter:

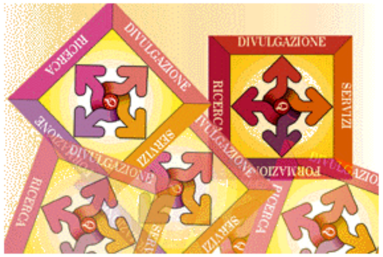
ICIE Cooperative Institute for the Innovation, Emilia Romagna, Italy

DESCRIPTION OF THE PROJECT

ICIE – ISTITUTO COOPERATIVO PER L'INNOVAZIONE

Cooperative Institute for the Innovation– supports and coordinates initiatives and activities within research, experimentation, technology transfer and organizational consulting, primarily in the construction, industrial and service sectors.

- **Development and implementation of applied research projects in cooperation with companies and research centers, national and international**
- **Consulting and technical assistance to local authorities for the construction of complex programs**
- **Business consultancy (organization and business skills)**



- ICIE has a structure expert and qualified, able to propose, plan, organize and manage even complex programs.

LaRCo icos

- ICIE is composed of engineers, architects, experts in the field of communication, economists, industrial experts, technical administration.

DESCRIPTION OF THE PROJECT

The application of the AHP in the REETI project aims to identify the best business plan for a sustainable energetic and environmental requalification of the existing receptive buildings.

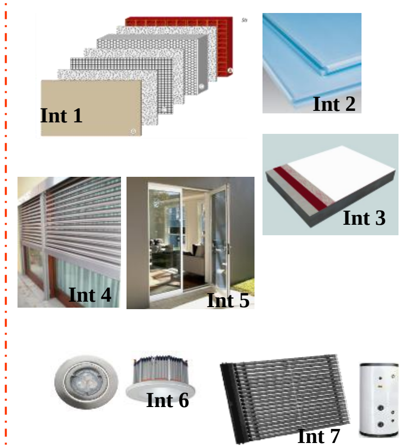
The demostrator receptive building is the

Hotel Antibes, in Italy

In particular, for a requalification directed to the efficiency, the safety and energetic-environmental sustainability for Antibes Hotel, the project proposes different kinds of innovative intervention, split into actions on the wrapping and installation:

TABLE II. PROPOSED INTERVENTIONS FOR HOTEL ANTIBES

Proposed Interventions	
Refer	*Interventions*
Intervention 1	Exterior Insulation and Finishing System
Intervention 2	Flat Roof Insulation
Intervention 3	Cool Roof
Intervention 4	External Heat Shielding
Intervention 5	Current Fixtures Substitution
Intervention 6	Bodies Illuminating Substitution with Highefficiency Lamp
Intervention 7	SunHeat for Domestic Hot Water (DHW)



DEFINITION OF THE HIERARCHICAL STRUCTURE OF THE PROBLEM

The first step of the AHP analysis is the definition of all elements of the project REETI that compose the hierarchical structure of the problem:

DECISION-MAKER the determination of the best decisionmaking solution is carried out by a single Decision-maker representative of the accommodation entrepreneur.

GOAL is the best Energetic and Environmental Requalification intervention of the Antibes Hotel in Riccione, Italy with the assignment of an energetic-environmental Certification.

"EAC Requalification"

DEFINITION OF THE HIERARCHICAL STRUCTURE OF THE PROBLEM

The first step of the AHP analysis is the definition of all elements of the project REETI that compose the hierarchical structure of the problem:

CRITERIA the analysis is conducted with respect to 9 Criteria, each of which divided into Sub-criteria

THE CRITERIA AND SUB-CRITERIA OF THE PROBLEM

Criteria of the problem	
Criteria	***Sub-criteria***
Cost	Investment Maintenance Management.
Consumption	Gas heating- DHW EE heating EE lighting DHW
Environmental Impact	CO2 TOE Architectural Integration
Indoor Comfort	Thermo Hygrometric Visual Acoustic Security
Attractiveness	Customer Partners Government
Productivity	Lifestyle Staff Blackout
Customer Satisfaction	Regular Customer New Customer
Profit Margin	Property value Room rate Extension service Tax cut
Risk	Preoperational Management

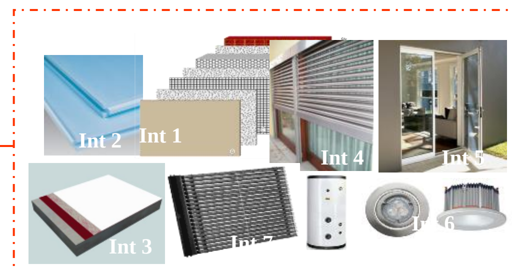
DEFINITION OF THE HIERARCHICAL STRUCTURE OF THE PROBLEM

The first step of the AHP analysis is the definition of all elements of the project REETI that compose the hierarchical structure of the problem:

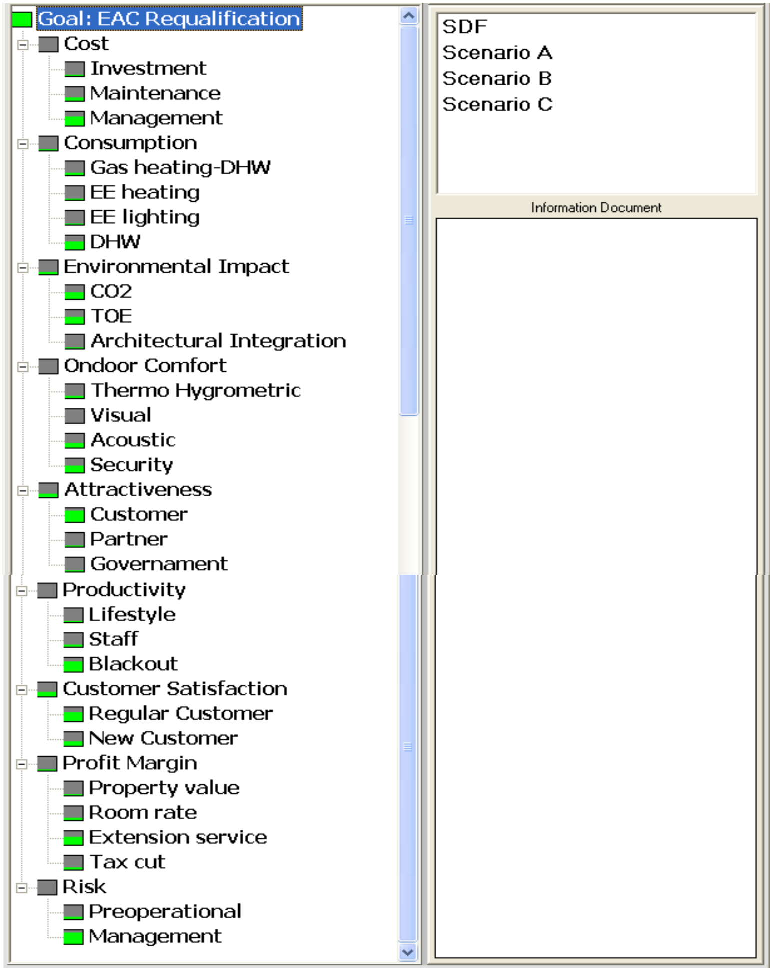
ALTERNATIVES The Alternatives considered in this problem are 4 and they differ based on interventions to be performed on the structure.

TABLE III. THE GOAL AND THE ALTERNATIVES OF THE PROBLEM

Elements	I and IV Hierarchy Level	
	Definition	*Acronym*
Goal	Energetic Environmental Certificated Requalification	EAC Requalification
Alternative 1	Stated Of Fact	SOF
Alternative 2	Intervention 4 Intervention 5 Intervention 6	Scenario A
Alternative 3	Intervention 3 Intervention 4 Intervention 5 Intervention 6	Scenario B
Alternative 4	Intervention 1 Intervention 2 Intervention 3 Intervention 4 Intervention 5 Intervention 6 Intervention 7	Scenario C



DEFINITION OF THE HIERARCHICAL STRUCTURE OF THE PROBLEM



Structure composed by 4 levels

by Expert Choice 11.5 software

PAIRWISE COMPARISON MATRICES AND WEIGHTS VECTORS

The pairwise comparisons are realized from top to bottom, so the first, the 9 Criteria are pairwise compared with respect to the Goal, then the Sub-criteria are compared with respect to each criterion and finally, the alternatives are compared with respect to Sub-criteria.

	Cost	Consumpti	Environme	Ondoor Co	Attractiven	Productivit	Customer S	Profit Marg	Risk
Cost		4,0	5,0	2,0	5,0	2,0	6,0	1,0	2,0
Consumption			1,0	5,0	3,0	1,0	3,0	2,0	4,0
Environmental Impact				5,0	1,0	6,0	2,0	1,0	6,0
Ondoor Comfort					6,0	3,0	4,0	3,0	4,0
Attractiveness						7,0	3,0	7,0	8,0
Productivity							6,0	2,0	6,0
Customer Satisfaction								9,0	9,0
Profit Margin									7,0
Risk	Incon: 0,09								

The criteria listed on the left are one by one compared with each criterion listed on top as to which one is more important with respect to the goal of selecting the best scenario.

For example, to the comparison between the criteria Customer Satisfaction and Profit Margins with respect to the Goal (the 7th-row, 8thcolumn) is assigned the value 9, expressing an extreme preference of the first criterium over the second.

Instead, the value 1 in the (2,3) position means that there is indifference between the criteria Consumption and Environmental Impact.

PAIRWISE COMPARISON MATRICES AND WEIGHTS VECTORS

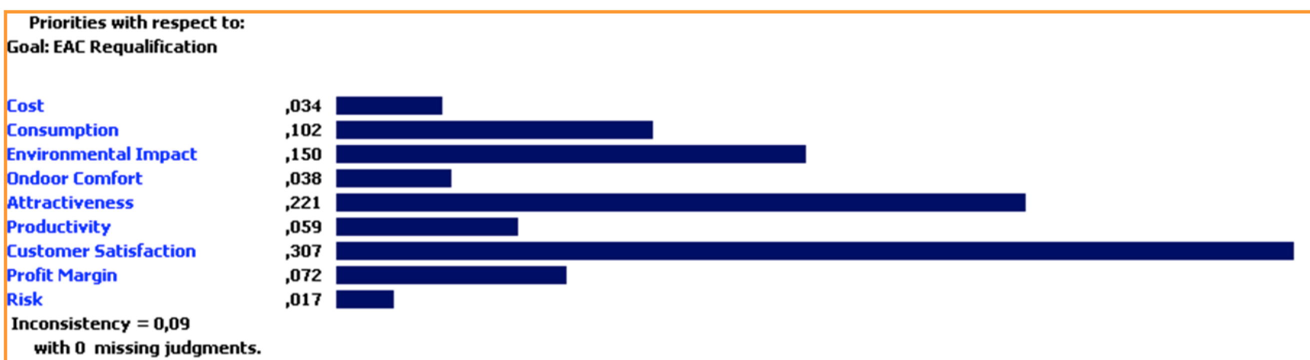
The judgements are entered by using the fundamental ***Scale of Saaty***

TABLE I. SAATY'S FUNDAMENTAL SCALE

The fundamental scale	
Intensity of importance	***Definition***
1	Equal importance
2	Weak or slight
3	Moderate importance
4	Moderate plus
5	Strong importance
6	Strong plus
7	Very strong or demonstrated importance
8	Very, very strong
9	Extreme importance
Reciprocals of above	If element *i* has one of nonzero numbers assigned to it when compared with element *j*, then *j* has the reciprocal value when compared with *i*

Once all judgements are entered, the software EC determines the local weight vector of the criteria, identifying the criterium with the highest weight (in this case, the Customer Satisfaction).

WEIGHTS VECTORS



The Consistency Index (CI), gives from the software, and the Random Index (RI), function of the size of the matrix, determine a Consistency Ratio (CR), that made the consistency check.

$$CR = \frac{CI}{RI}$$

	Cost	Consumpti	Environme	Ondoor Coi	Attractiven	Productivit	Customer S	Profit Marg	Risk
Cost		4,0	5,0	2,0	5,0	2,0	6,0	1,0	2,0
Consumption			1,0	5,0	3,0	1,0	3,0	2,0	4,0
Environmental Impact				5,0	1,0	6,0	2,0	1,0	6,0
Ondoor Comfort					6,0	3,0	4,0	3,0	4,0
Attractiveness						7,0	3,0	7,0	8,0
Productivity							6,0	2,0	6,0
Customer Satisfaction								9,0	9,0
Profit Margin									7,0
Risk	Incon: 0,09								

CI

Acceptable limit

CR < 10%

In the following 9X9 matrix the CR is lower than the acceptable limit imposed by Saaty

$$CR = \frac{0,09}{1,45} = 0,06$$

THE JUDGEMENTS ENTERED ARE CONSIDERED RELIABLE

CONSISTENCY CHECK

For the other matrices of the lower levels, the consistency check leads to validate the judgements entered, because the CR of all matrices are below the max acceptable limit fixed by Saaty.

TABLE V. CHECK OF CONSISTENCY OF THE PAIRWISE COMPARISON MATRICES OF SUB-CRITERIA WITH RESPECT TO CRITERIA

Criteria	Check of Consistency of the III level matrices with respect to the Criteria	
	CI	***CR***
Cost	0.00	0.00
Consumption	0.04	0.04
Environmental Impact	0.00	0.00
Indoor Comfort	0.08	0.09
Attractiveness	0.00	0.00
Productivity	0.01	0.02
Customer Satisfaction	0.00	0.00
Profit Margin	0.04	0.04
Risk	0.00	0.00

Acceptable limit

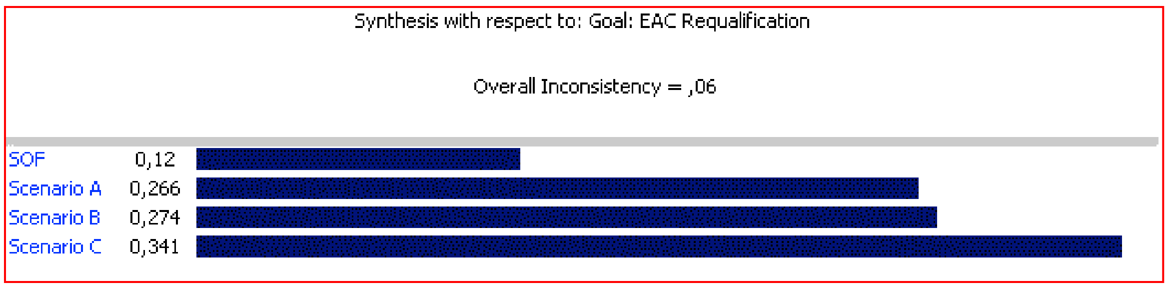
CR < 10%

THE JUDGEMENTS ENTERED ARE CONSIDERED RELIABLE

RANKING OF THE ALTERNATIVES

The overall priorities of the alternatives obtained by aggregating the local priorities provide the ranking of the alternatives.

The values of the global vector of the alternatives are:



The best business plan for a sustainable energetic and environmental requalification of the Antibes Hotel is……

the best solution

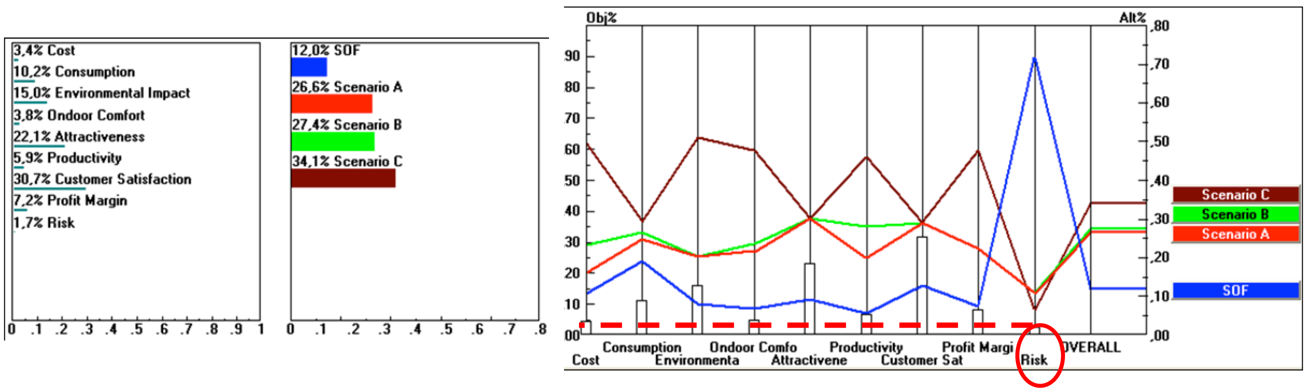
"SCENARIO C"

SENSITIVITY ANALYSIS

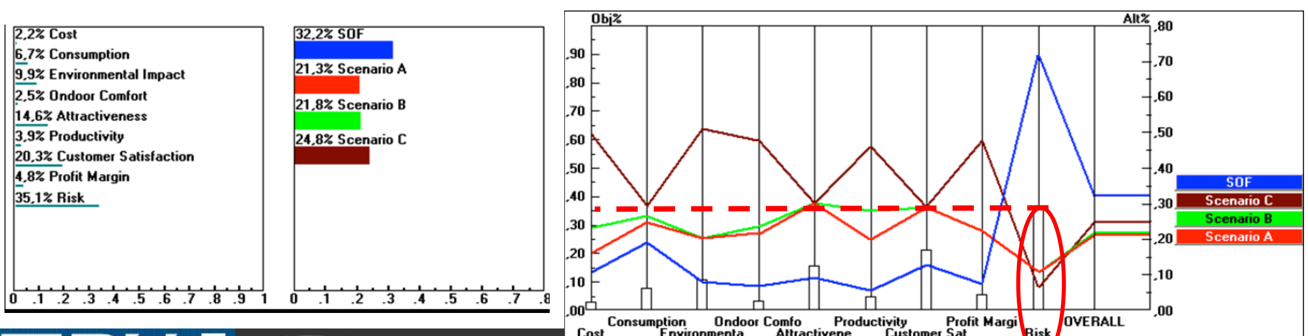
A **Sensitivity Analysis** is conducted in order to monitor the robustness of the preference ranking among the alternative scenarios to changes in the criteria weights.

For the analysis, we have chosen to change the weight of the criterium "Risk", because is the only criterium for which there is a change of solution:

Performance Graphic



The Performance Graphic shows the rank reversal, by increasing the weight of the criterium "Risk" from 0.017 to 0.35.



The most preferred alternative is "SOF"

Cavallo B., Squillante M.

SENSITIVITY ANALYSIS

The most preferred alternative "SOF" → ***NON ACCEPTABLE and TRIVIAL***

WHY ?

1. The other criteria are more important than criterium Risk:

By increasing the weight of the criterium "Risk", the importance of all other criteria decreases automatically (such as Customer Satisfaction, Environmental Impact and Attractiveness, at the beginning considered more important than the risk by the decision maker), because the weights are normalized to 1.

2. Every investment generates risk:

Because the alternative SOF is a non-intervention, if you give more importance to the Risk criterium, of course, the solution becomes the scene SOF.

SENSITIVITY ANALYSIS

FINDINGS

The best solution for the Energetic and Environmental Certified Requalification of the Antibes Hotel in Riccione is the alternative 4

THE BEST SOLUTION

"SCENARIO C"

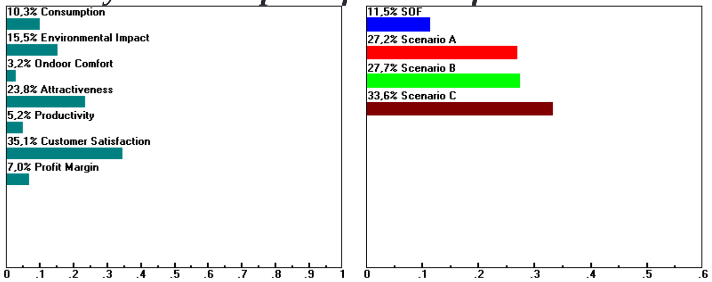
COMMENTS

Is there a combination of criteria weights that gives "Scenario A or B" as best solution?

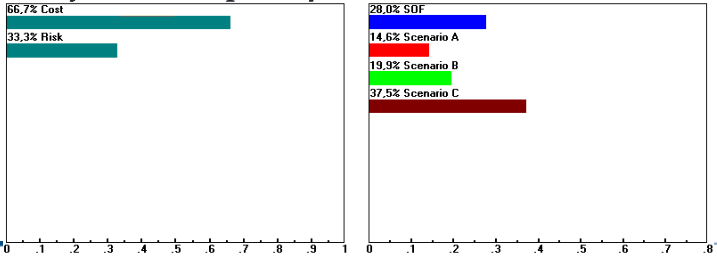
YES!

If we perform the Benefit/Cost Analysis by separating the Cost and Risk criteria from the benefits, is obtained as the best solution the scenario A.

Dynamic Graphic of the benefits criteria



Dynamic Graphic of the Cost and Risk criteria



BENEFITS/COSTS ANALYSIS

The Best solution → SCENARIO A

SCENARIO SOF	0,410714286
SCENARIO A	1,863013699
SCENARIO B	1,391959799
SCENARIO C	0,896

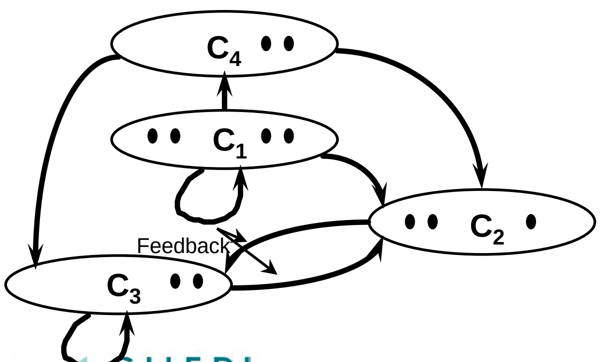
FUTURE WORKS

Only 1 decision maker

For simplicity, but it can be implemented by a decision-maker team

Some other MCDM method could be used

For the presence of dependence between the alternatives, would be appropriate to apply the ANP and compare these results with those obtained using the AHP.



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Optimization of a Portfolio

Markowitz theory
basic principle

To identify a combination of securities to minimize risk and maximize overall performance, with a classical mean-variance approach.

↓

The model is based on estimates of future parameters.

Goal

TO BUILD A MODEL THAT ALLOWS:

- To limit forecast activities
- To minimize subjectivity

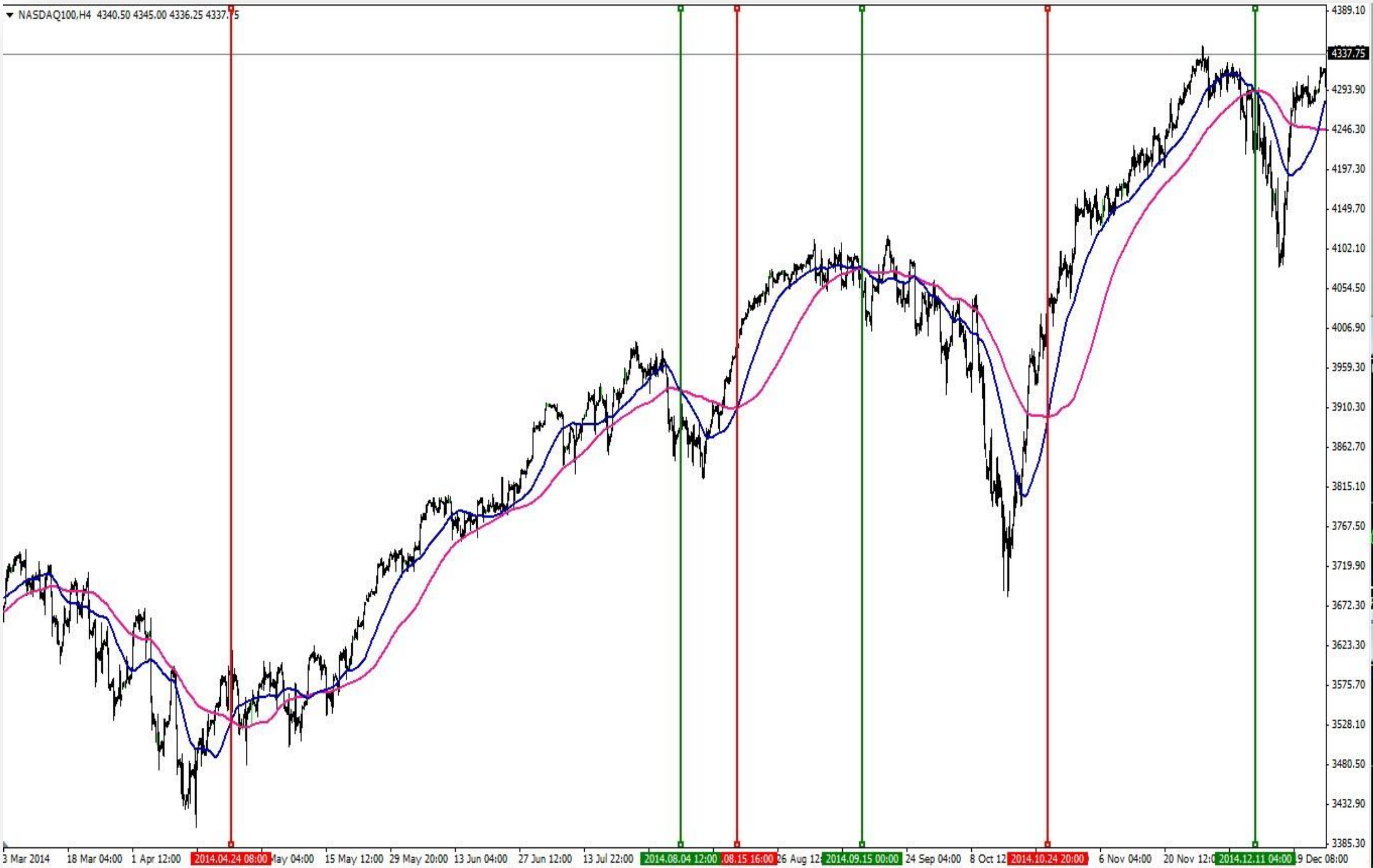
↓

A MIX OF:

Quantitative Methods

Qualitative Methods

NASDAQ100


NASDAQ100,H4 4340.50 4345.00 4336.25 4337.75
4389.10
4337.75
4293.90
4246.30
4197.30
4149.70
4102.10
4054.50
4006.90
3959.30
3910.30
3862.70
3815.10
3767.50
3719.90
3672.30
3623.30
3575.70
3528.10
3480.50
3432.90
3385.30
3 Mar 2014
18 Mar 04:00
1 Apr 12:00
2014.04.24 08:00
May 04:00
15 May 12:00
29 May 20:00
13 Jun 04:00
27 Jun 12:00
13 Jul 22:00
2014.08.04 12:00
.08.15 16:00
26 Aug 12
2014.09.15 00:00
24 Sep 04:00
8 Oct 12
2014.10.24 20:00
6 Nov 04:00
20 Nov 12:0
2014.12.11 04:00
9 Dec 08:00

Moving Average

$$MM_{n,t}(x) = \frac{\sum_{i=0}^{N-1} x_i}{N}$$

$$If\ \ MM_{n,t}(x) > MM_{N,t}(x) \text{ and } MM_{n,t-1}(x) \leq MM_{N,t-1} \text{ then BUY}$$

$$If\ \ MM_{n,t}(x) < MM_{N,t}(x) \text{ and } MM_{n,t-1}(x) \geq MM_{N,t-1} \text{ then SELL}$$

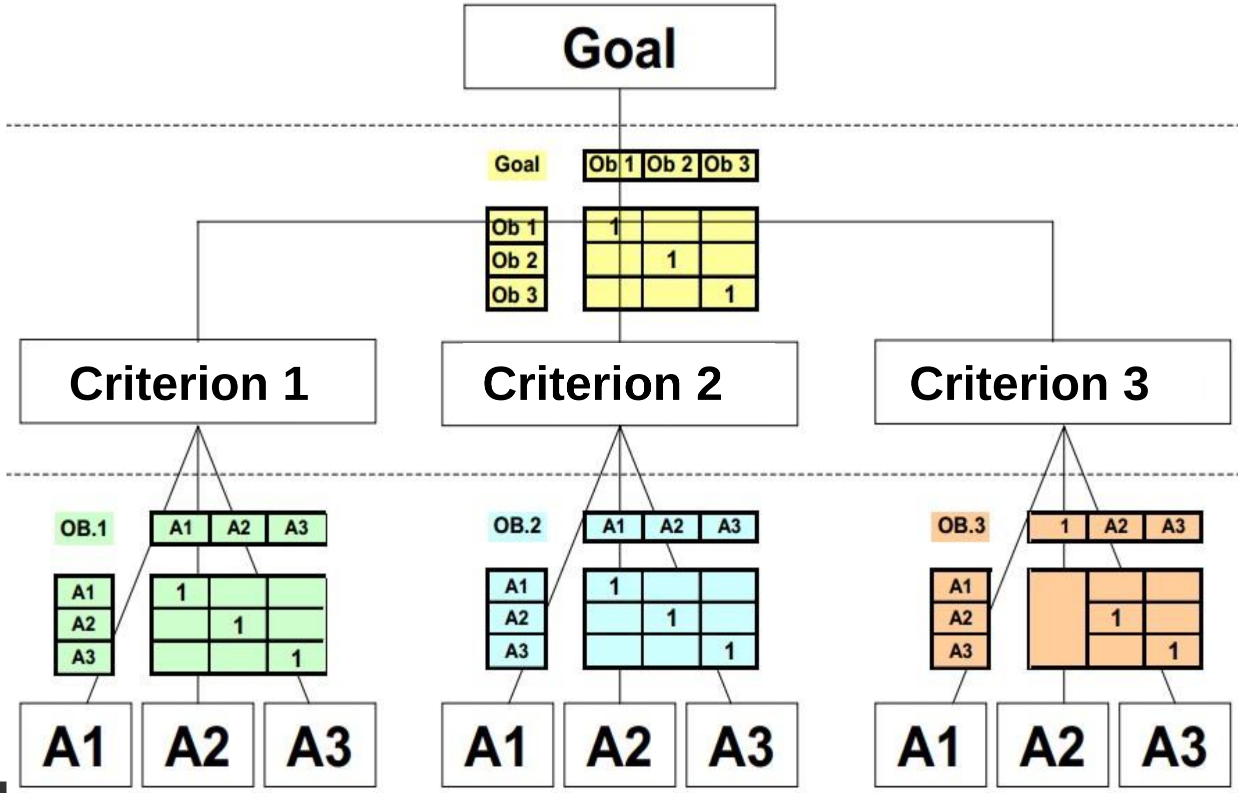
Sample analyzed

Quantitative Analysis

Company/Session	I Period	II Period	III Period
Apple	+ΔP	+ΔP	+ΔP
Facebook	+ΔP	+ΔP	-ΔP
Intel	+ΔP	+ΔP	+ΔP
Yahoo	+ΔP	+ΔP	+ΔP
Amazon	-ΔP	-ΔP	+ΔP
Google	+ΔP	+ΔP	-ΔP
Vodafone	-ΔP	-ΔP	+ΔP
Ebay	+ΔP	-ΔP	+ΔP

Portfolio Selection

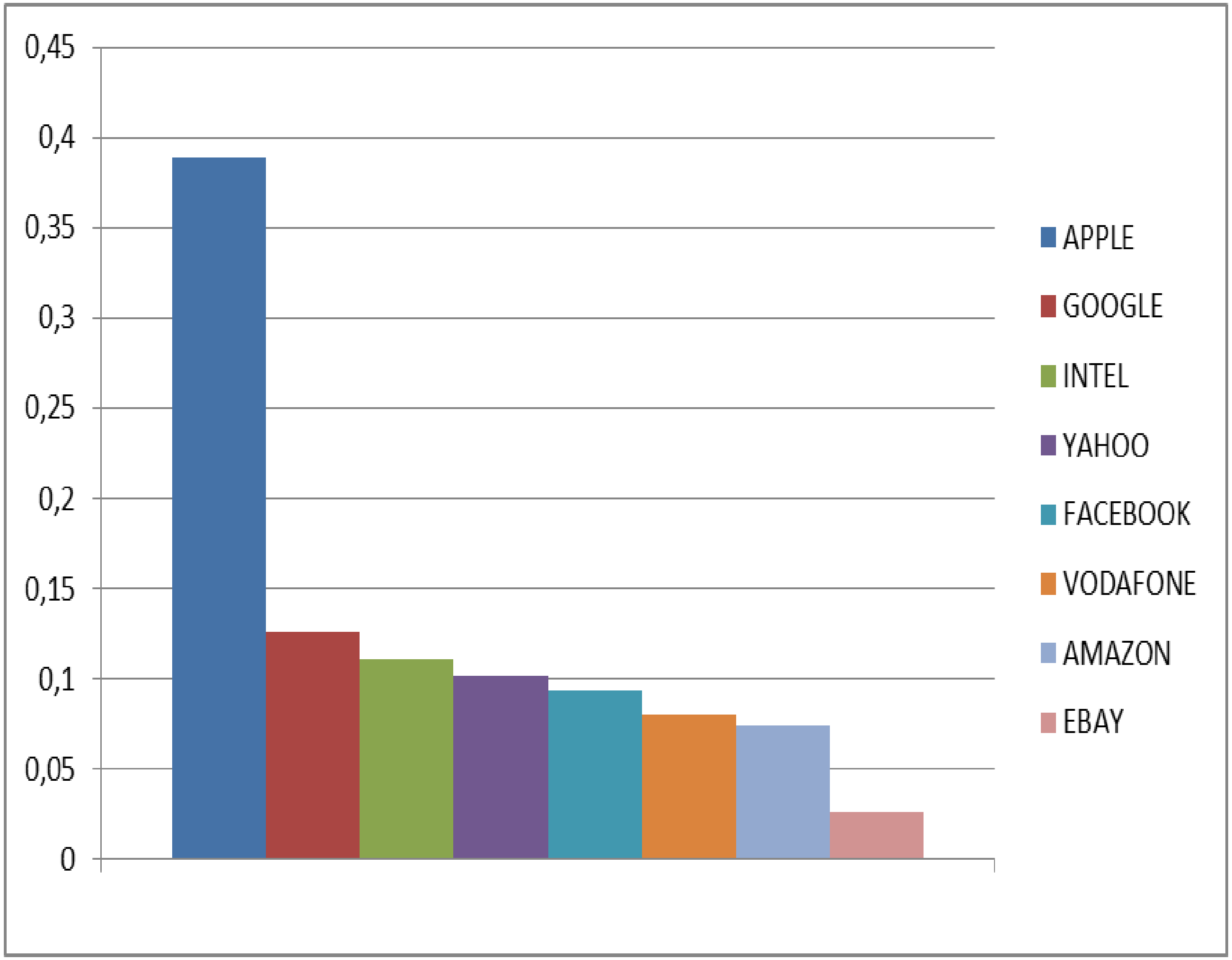
Qualitative Analysis: AHP



AHP for trading apps

- Goal: max financial returns
  - Financial criteria
    - Capital
    - Net income
    - Revenues
  - Indices
    - Stock prices
    - Market capitalization
    - Stock prices changes
  - Management efficiency
    - Business policy
    - Business management
    - Brand loyalty
  - Technological criteria
    - Outputs
    - Innovative skills
    - Plants value

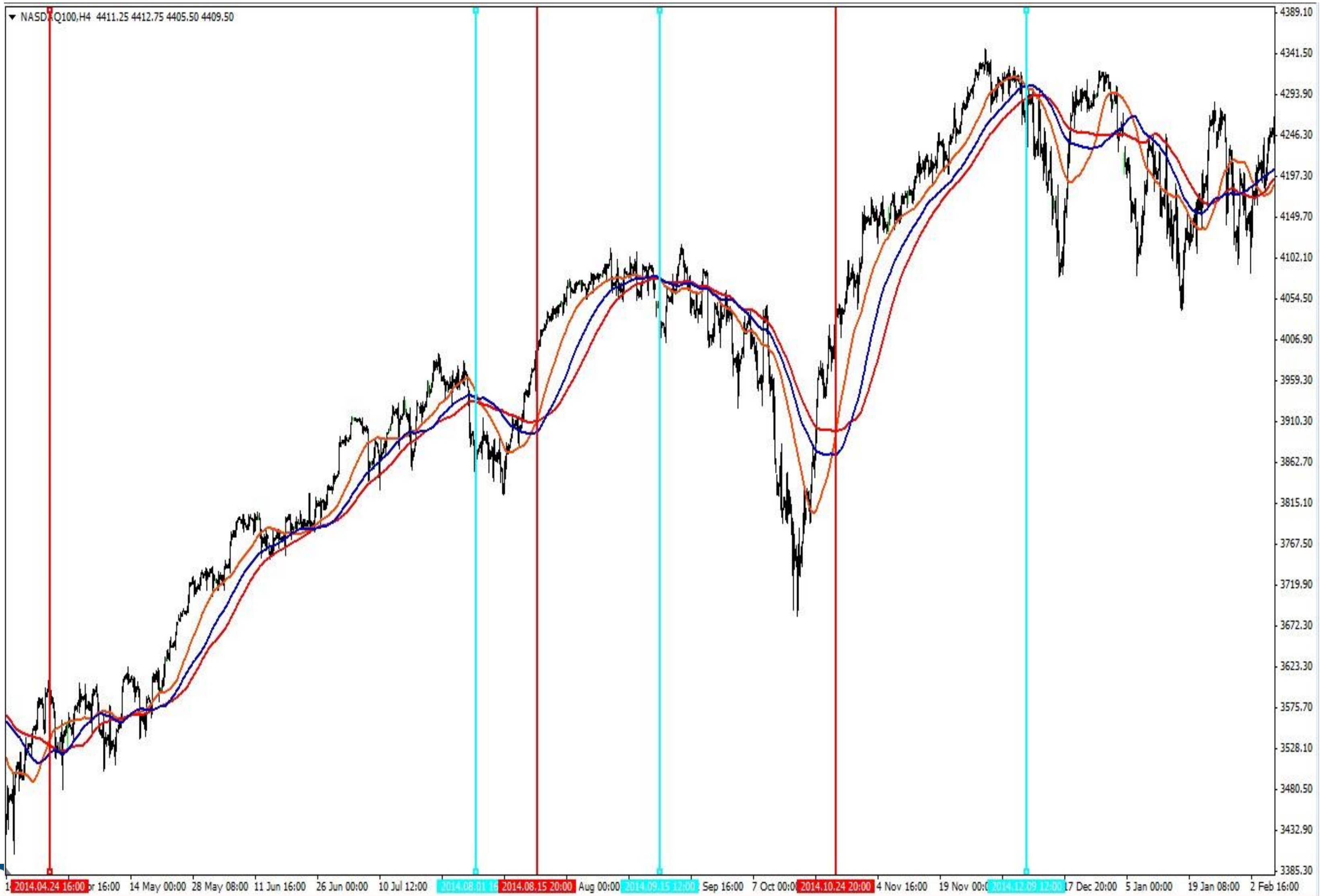
AHP: Result


0,45
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
APPLE
GOOGLE
INTEL
YAHOO
FACEBOOK
VODAFONE
AMAZON
EBAY

Portfolio Optimization

ΔP	I period	II period	III period
APPLE	ΔP +	ΔP +	ΔP +
GOOGLE	ΔP +	ΔP +	ΔP -
INTEL	ΔP +	ΔP +	ΔP +
YAHOO	ΔP +	ΔP +	ΔP +

Portfolio Maximization



Comparison

Optimized Portfolio

ΔP	I periodo	II periodo	III periodo
APPLE	21.48	3.65	4.51
GOOGLE	47.99	7.87	-103.2
INTEL	7.3	0.37	2.75
YAHOO	1.9	6.08	6.32

Capital Gain: 7,02 $ for stock

Maximized Portfolio

ΔP	I periodo	II periodo	III periodo
APPLE	15.02	3.65	8.9
GOOGLE	40.91	7.87	-6.40
INTEL	6.99	0.37	3.72
YAHOO	0.38	6.08	7.01

Capital Gain: 94,5 $ for stock

Start Up Project: TrOFIS

- M. Squillante, B. Cavallo, F. Ferrara, M.G. Olivieri, V. Ventre

- Start Up Project TrOFIS: Trading Operational Financial Integrated System (Qualitative and Quantitative analysis)
  - presented at the contest START-CUP CAMPANIA
  - http://www.startcupcampania.unina.it (innovative spin-off and start-up projects)

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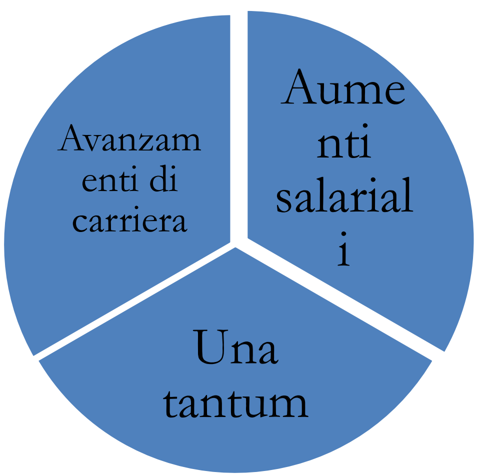
Management of human resources

Azienda leader nel settore dell'alta tecnologia.

La *Salary Review* è uno dei processi di rewarding a supporto della politica di valorizzazione delle risorse umane.



- Si pone come raccordo tra la pianificazione e la valorizzazione delle risorse umane.
- La sua implementazione è affidata alla funzione Risorse Umane
- Si sostanzia nella distribuzione annuale di un ammontare monetario pari al 15% del Budget RU
- Tocca aspetti sensibilmente connessi a motivazione e conflitto.

Management of human resources

L'assegnazione al *Work Level* e la valutazione delle performance costituiscono momenti fondamentali per l'accesso alle politiche di *Salary Reward* definite a livello globale.

La strategia di rewarding dell'azienda è regolata da tre componenti principali:

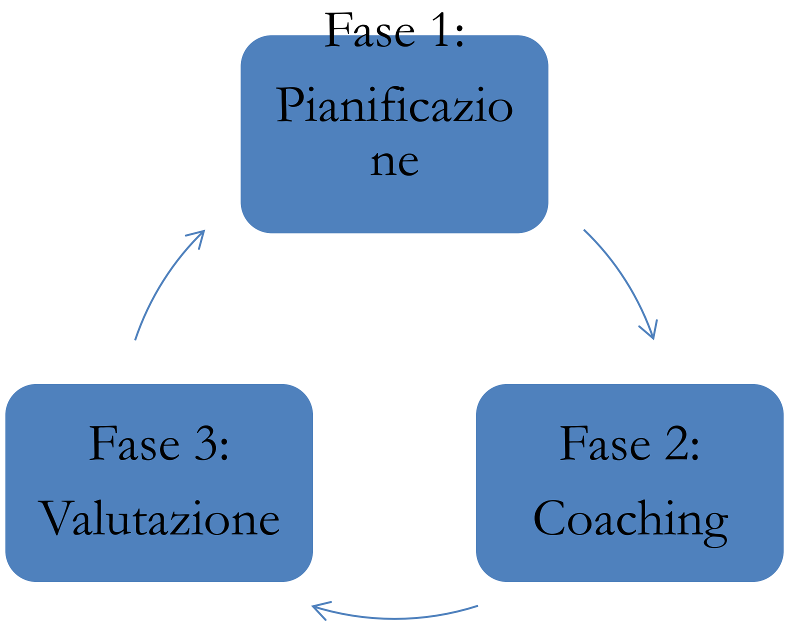
1. Le performance individuali, con riferimento agli ultimi due anni;
2. Il peso della posizione
3. I riferimenti allo scenario locale.

Le dimensioni considerate sono:

- *Know-how*
- *Problem solving*
- *Accountability*

Management of human resources

La valutazione delle prestazioni è uno degli strumenti fondamentali per gestire lo sviluppo delle risorse umane. Tale processo permette di definire obiettivi e percorsi di crescita individuali. Perché sia efficace, è necessario che gli obiettivi siano condivisi.



Comunicazione tra capi e membri del team

⇕

Responsabilizzazione delle risorse umane

⇕

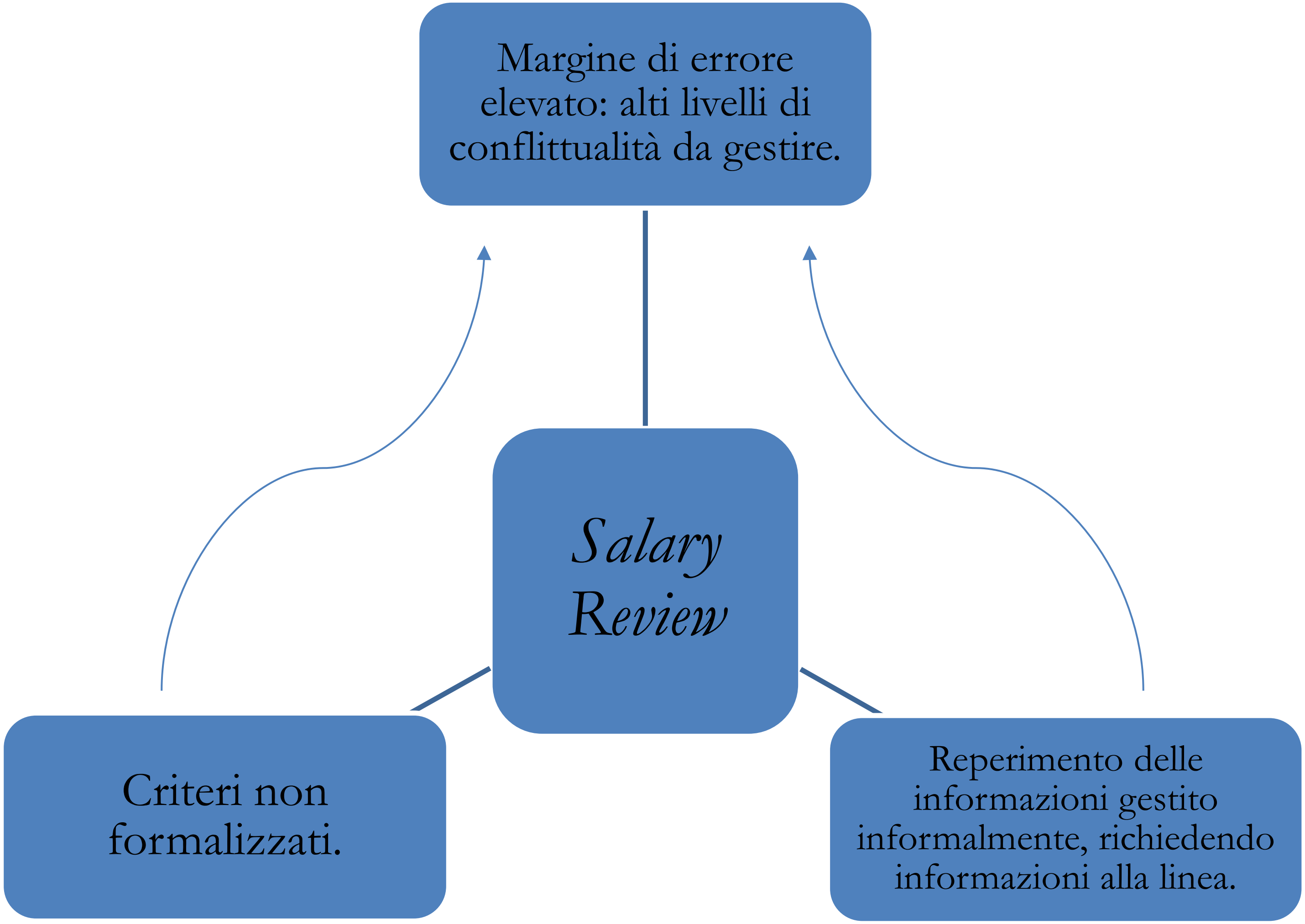
Condivisione e allineamento degli obiettivi

Management of human resources

Il driver principale del processo è da ricercarsi nella comunicazione tra capo e collaboratore, che assicura la costruzione di percorsi mirati di sviluppo delle competenze (copertura dei gap emersi), favorendo la partecipazione a corsi di formazione, coaching e inserimento in specifici progetti aziendali.

Rende, inoltre, possibile l'individuazione di *best performers*, per i quali sono previsti percorsi di sviluppo ad hoc e forme di *compensation* coerenti con il livello di prestazione espresso.

Management of human resources



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